THERMAL ENERGY

FEBRUARY 27, 2022

Theory recap

Relating internal kinetic energy and temperature. Last time we discussed temperature. We learned that higher temperature corresponds to larger internal kinetic energy and today we will make this relation precise. Kelvin scale is especially convenient for relating temperature and average internal kinetic energy of atoms and molecules. This relation is as follows:

$$E_{kin} = \frac{3}{2}kT$$

,

where T is temperature in Kelvins and $k = 1.38 \cdot 10^{-23}$ J/K is called the Boltzmann constant. Boltzmann constant is the coefficient between average internal kinetic energy of atoms and molecules and temperature. It is a very small number, so kinetic energies of atoms and molecules are very small. This is because their mass is very tiny. Actually, knowing the mass of atoms and molecules we could use the relation between temperature and average kinetic energy to estimate what velocities do they have at the temperature we are used to.

As an example let us take an oxygen molecule O_2, which has mass $m_{O_2} = 5.3 \cdot 10^{-26}$ kg. What is the average speed of such a molecule at temperature 300 K? (this temperature is close to usual room temperature and is often used in estimates for convenience)

$$\frac{m_{O_2}v^2}{2} = \frac{3}{2}kT \Longrightarrow v^2 = \frac{3kT}{m_{O_2}} \Longrightarrow v = \sqrt{\frac{3kT}{m_{O_2}}} = 484\ m/s$$

Compared to a speed of commercial airplane (about 250 m/s) we see that oxygen molecules at room temperature move almost two times faster! But normally they don't travel far because due to collisions they change direction of motion very often.

Internal energy, specific heat capacity. Actually, internal kinetic energy is not the only kind of internal energy - there is also internal potential energy which we only mention but not discuss in detail. Together, internal kinetic and potential energy comprise **internal energy**.

Internal kinetic energy grows with temperature and we know exactly how does it grow on the microscopic level of individual atoms and molecules (as explained above). Internal potential energy also grows with temperature but how exactly it grows on the microscopic level depends on the structure of an object. We are not interested in the object structure at the microscopic level, so instead we describe internal energy **macroscopically**: at the size scale of ordinary objects, not atoms or molecules. Thermal properties of every material are characterized by a constant known as **specific heat capacity**, denoted by c. It relates change in internal energy of an object made out of this material with the change in its temperature as follows:

$$\Delta E_{int} = cm\Delta t.$$

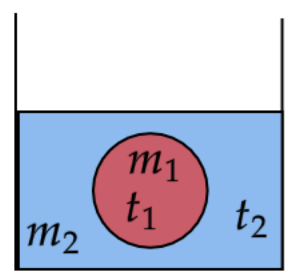
where ΔE_{int} is the change in internal energy of an object, Δt is change in temperature and m is the objects' mass. First important feature of this formula is that internal energy grows as temperature grows. It is also important that for an object of larger mass more energy is needed to change its temperature by the same amount.

Specific heat capacity is a property of the material which is measured experimentally. Units of specific heat capacity are J/(kg·°C). For example, for water $c = 4200$ J/(kg·°C). For iron it is $c = 460$ J/(kg·°C). Basically, specific heat capacity tells us how much energy in Joules is needed to heat 1 kg of a substance by 1 °C.

Heat transfer Having talked about internal energy, let us now talk about how it could be changed. In other words, how do we make an object hotter? A common way is heat transfer from another hot object. We know that if two objects of different temperature are brought into contact their temperature tends to become equal as time passes. For example if you put an apple in the refrigerator the apple will soon become cold, because it is in contact with cold air and surface in the refrigerator. The fact that temperatures of objects in contact become equal is referred to as reaching thermal equilibrium. Our understanding of temperature would allow us to understand the basic underlying mechanism of reaching thermal equilibrium. Remember that temperature is a measure of internal kinetic energy. So if one object has higher temperature than the other, it means that kinetic energy of atoms and molecules in the first object is larger than in the second object. When these objects are brought into contact, their atoms and molecules collide with each other and during these collisions the ones with higher kinetic energy give their extra energy to the ones with lower kinetic energy. As a result, kinetic energies of atoms and molecules in both objects tend to become the same. Therefore, temperatures tend to become the same.

The situation when internal energy is transferred from one object to another is known as heat transfer. In practice we could use the formulas for internal energy change in order to solve various problems. In particular, a common task is to find the equilibrium temperature after two objects of particular temperatures are brought into contact.

Let us look at such an example: we take a piece of iron of mass $m_2 = 2$ kg which has temperature $t_2 = 100°$ C and put it in a container with $m_1 = 1$ kg of water at temperature $t_2 = 20°$ C. What temperature will water and iron reach after coming to thermal equilibrium? As often happens in such problems, we will assume that the container with water is well insulated from the environment, so no heat could flow between our system of water + iron and the environment.



Let us call the final temperature of water and iron t_3 - that is what we need to find. We would also need specific heat capacities of water and iron which we will call c_1 and c_2 correspondingly (their numerical values were provided above). The most important principle we should use is energy conservation law applied to internal energy. Since the system is insulated and no mechanical energy is dissipated and no work is being done, total internal energy must stay the same. This means that the change in total internal energy is 0:

$$\Delta E_{tot\ int} = \Delta E_1 + \Delta E_2 = 0$$

.

Here ΔE_1 is change of internal energy of water and ΔE_2 is the change of internal energy of iron. Using our general formulas we get:

$$\Delta E_1 = c_1 m_1 \Delta t_1 = c_1 m_1 (t_3 - t_1);\ \Delta E_2 = c_2 m_2 \Delta t_2 = c_2 m_2 (t_3 - t_2)$$

where $\Delta t_1, \Delta t_2$ are changes of temperature of water and iron correspondingly and are found as final temperature minus initial temperature. Plugging this back to our energy conservation condition we get

$$c_1 m_1 (t_3 - t_1) + c_2 m_2 (t_3 - t_2) = 0$$

Now the physics part of the problem has ended and we just need to mathematically solve the equation for t_3. To do it we gather all the terms with the unknown t_3 on the left hand side of the equation and all the terms without t_3 on the right hand side:

$$c_1 m_1 t_3 + c_2 m_2 t_3 = c_1 m_1 t_1 + c_2 m_2 t_2 \Longrightarrow t_3 = \frac{c_1 m_1 t_1 + c_2 m_2 t_2}{c_1 m_1 + c_2 m_2}$$

.

Now we just need to plug in the numbers:

$$t_3 = \frac{4200 \cdot 1 \cdot 20 + 460 \cdot 2 \cdot 100}{460 \cdot 2 + 4200 \cdot 1} {}^\circ\mathrm{C} = 34^\circ\mathrm{C}$$

.

We see that the answer is 34° C, quite close to the initial temperature of the water. So water temperature did not change much while iron temperature changed a lot. This is so because specific heat capacity of water is much bigger than specific heat capacity of iron.

Homework

1. Temperature in the room is increased from 15° C to 39° C. How many times did the average kinetic energy of the air molecules increase?

2. Mr. X does not like when his morning coffee is too hot so he adds some cold milk to it. Initially the coffee is at boiling temperature (100°C) and milk is just out of the fridge (10 ° C). How much milk does Mr. X have to add to 150 g of coffee in order for the mixture to have temperature 65° C? You may assume that both coffee and milk have the same specific heat capacity as water.

3. A 500 gram cube of lead is heated from 20 ° C to 80 ° C. How much energy was required to heat the lead? The specific heat capacity of lead is 160 J/kg·°C.

***4.** Some object with initial temperature $t_1 = 100^\circ$ C is put in a glass with water with initial temperature $t_2 = 10^\circ$ C. After some time thermal equilibrium established and temperature became $t = 40^\circ$. Then another object, completely the same as the first one and also with initial temperature $t_1 = 100^\circ$ C was put in the same glass with the first object still in it. What will the resulting temperature t' be?