# GAS LAWS

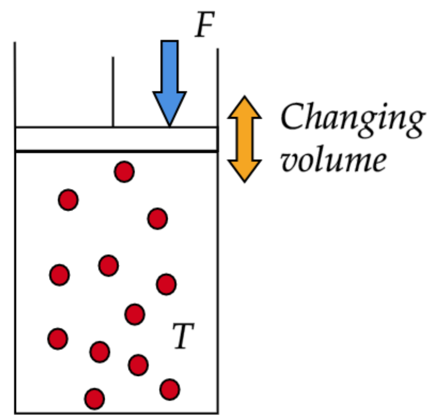
MARCH 13, 2022

## Theory recap

Last time we discussed pressure. We did it in order to be able to discuss the behavior of gases more generally. As we already mentioned, a gas is characterized by its temperature $T$, volume $V$ and pressure $p$. Temperature is the measure of internal kinetic energy of atoms or molecules comprising the gas. Volume is the measure of how much space does the gas take up. Pressure is the measure of force exerted by the gas on the walls of the container.

Amazingly, these three quantities are not independent. Our today's class is going to be about relations between these quantities.

First, let us discuss what we are actually going to study. Let us consider a container with a variable volume. The easiest way to achieve it is to attach a piston, like in a syringe but airtight so no gas could escape. Then by moving the piston back and forth we could change the volume of the gas. Pressure of the gas is trying to push the piston upwards, and in order to compensate for it we have to push is downwards. If pressure of gas is $p$ and area of the piston is $A$, the force with which we have to push is




$$F = pA.$$

By changing this force we would change the pressure of the gas - the system will only come to equilibrium when forces acting on the piston are balanced. Finally, we could heat or cool the gas by bringing it to contact with hot or cold objects. For clarity we can think that when we heat the gas we put it next to a burner and when we cool the gas we put the container into a bucket with ice. We would assume that all gas has the same temperature at every moment of time (neglecting that the part closer to a heat source in reality heats a little faster). Temperature of the gas could be measured by a thermometer inside the container.

Now we have the tools to explore gas laws. There are three laws and they have similar structure: in each of them on of the basic quantities $T, V, p$ is kept constant and a relation between the other two is studied. Today we will study just one of them with the others to follow.

**Boyle's law.** First let us keep $T$ constant. We could achieve it by keeping the environment at constant temperature and doing all changes in volume or pressure slowly enough so at every moment the gas is in thermal equilibrium with the environment and its' temperature is constant. This kind of process is called **isothermal**. For example we could slowly put additional blocks on the piston to change the pressure and look at how volume of the gas changes. This way Boyle's law was discovered:

$$pV = const \quad \text{for } T = const$$

which means that at constant temperature pressure and volume are inversely proportional to each other. If pressure is increased, the volume decreases and vice versa. Another way of writing Boyle's law is

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Here $p_1$ and $V_1$ are respectively pressure and volume at some state of the gas and $p_2$ and $V_2$ are at some other state with the same temperature. Let us consider an example: say that some gas is in a vessel with piston and initially volume is 100 cm$^3$ and pressure is 1000 Pa. Then pressure is slowly increased to 2000 Pa and temperature is kept the same. What will the volume be? Using Boyle's law we get:

$$V_2 = V_1 \cdot \frac{p_1}{p_2} = 100\ cm^3 \cdot \frac{1000}{2000} = 50\ cm^3$$

Of course, we see that since we increased pressure in two times, volumes became two times smaller. We could ask the same question (what will the volume be) about pressure 3000 Pa, 4000 Pa, etc. We could ask what the volume will be if instead of increasing pressure we decreased it to 800 Pa, 600 Pa, 500 Pa, etc. Then we could take all such pairs of pressure and volume and represent them as points in $p - V$ plane. Through these points we could draw a curve known as isotherm - which from Greek means "constant temperature". This curve represents pressure as a function of volume: since $pV = const$, $p = \dfrac{const}{V}$, so pressure is a function of volume.

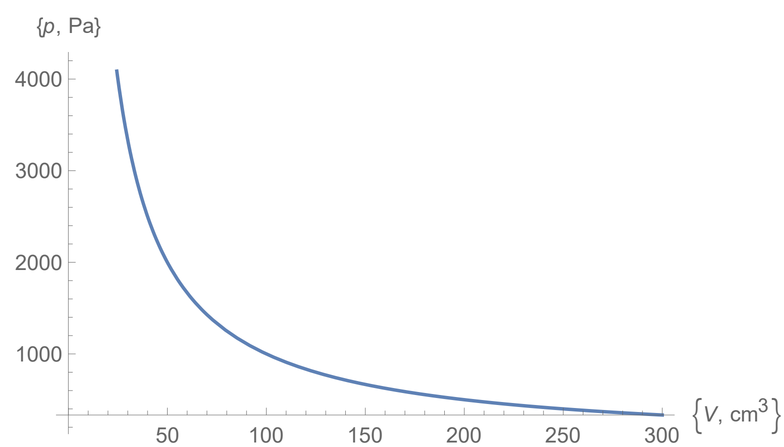



FIGURE 1. The curve shown is isotherm - it shows all possible combinations of pressure and volume for some given temperature. If you know the pressure, just look at the curve - and you will know the volume. In mathematics the curve of such shape is known as hyperbola

## Homework

**1.** A cylinder with a piston is filled with gas. The pressure inside is 1000 Pa. We push the piston inside the cylinder and decrease the volume of the gas two times. Find the new pressure if the temperature of the cylinder is kept constant.

**2.** A cylinder with a piston is filled with gas at pressure 100 000 Pa. Again, the temperature of the cylinder is kept constant. The pressure inside the cylinder is equal to the pressure outside the cylinder, so the piston does not move. The volume of the gas inside the cylinder is 1000 $\mathrm{cm}^3$. We put a 10 kg stone on the piston. The piston moves down and stops, compressing the gas in the cylinder. Find the new volume of the gas if the area of the piston is 10 $\mathrm{cm}^2$. (To solve this problem you have to remember what the pressure is and how we calculate it).

***3.** In order to make volume of a gas in a cylinder with a piston 3 times smaller one needs to place a block of mass $m$ on the piston. What is mass of another block one needs to add to this one so that volume further becomes 4 times smaller? Temperature is kept constant.