COMBINED GAS LAW.

MARCH 27, 2022

THEORY RECAP

Last time recap. Last time we discussed Gay-Lussac's law in addition to Boyle's law discussed earlier. To summarize, they read

$$pV = const \text{ for } T = const$$

$$p/T = const \text{ for } V = const$$

Having discussed the cases of constant temperature and constant volume, the last remaining case (and the corresponding gas law) deals with the constant pressure.

Charle's law. In practice we could easily keep the pressure constant by applying a constant force on the piston. We do not even have to apply force ourselves - atmospheric pressure is always acting on the piston from the outside. While keeping constant pressure we could heat the gas and measure how does it affect the gas volume. This way Charles's law was discovered (by French physicist and engineer Jacques Charles who was also the creator and first pilot of a manned hydrogen balloon). Charles's law says that volume changes proportionally to the temperature measured in Kelvins. If we compare gas at two states such that at one of them it has volume V_1 and temperature T_1 and in the other its' volume and temperature are V_2 and T_2, Charles's law reads:

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}.$$

An equivalent way of writing Charle's law is:

$$\frac{V}{T} = const \text{ for } p = const.$$

For example, if at 300 K some gas had volume 300 cm^3 then at 400 K and at the same pressure it will have volume 400 cm^3, at 500 K volume will become 500 cm^3 etc. Let us try to visualize this law on a plot. The axes are temperature T versus volume V. T proportional to V corresponds to a straight line on this plot.

Note that this line goes through the point $(0, 0)$ so this plot predicts that at 0 K the gas will have zero volume. In reality, this is nonsense - nothing could have zero volume. The truth is, Charles's law and other gas laws are established for an **ideal gas** which is only an approximation to any real gas. In an ideal gas there is no interaction between its' molecules at all. In a real gas there is always some interaction but if the gas is rarefied enough it could be neglected. So the ideal gas is a good approximation for a gas of small density. However as gas becomes denser approximation by an ideal gas becomes worse. For a very low temperature and therefore small volume it is not a good approximation and Charles's law (and other laws) do not hold anymore.

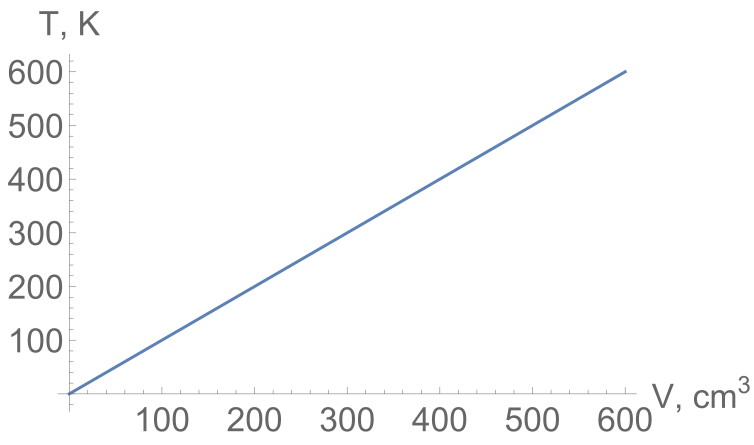

FIGURE 1. The line corresponds to constant pressure. Note that the temperature is in Kelvins

Combined gas law. We have discussed three gas laws and in every one of them there is one parameter that we keep fixed. But what if we want to change all three parameters of the gas? Say we start from gas having pressure p_1, volume V_1 and temperature T_1. We engage the gas in some process so that as the result it has pressure p_2, volume V_2 and temperature T_2. Our question is, could these 6 parameters be related to each other somehow? The answer is yes!

We could actually derive the relation between the aforementioned 6 parameters from the gas laws we have already established. But we have to be clever about it since our gas laws would only work if one of the three parameters is fixed. Luckily, nothing prevents us from reaching the final state in two steps: one at fixed temperature and another at fixed volume (see figure 1). We will do the following to thegas: first, we keep temperature fixed and change volume from V_1 to V_2. Pressure then also changes to some p_3 and as we know from Boyle's law:

$$p_3 V_2 = p_1 V_1 \implies p_3 = \frac{p_1 V_1}{V_2}.$$

After we are done with this, we could keep the volume fixed and now change temperature from T_1 to T_2. Pressure also changes from p_3 to p_2. As we know from Charles's law:

$$\frac{p_2}{T_2} = \frac{p_3}{T_1} \implies \frac{p_2}{T_2} = \frac{p_1 V_1}{V_2 T_1} \implies \frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}.$$

In the second step we have used the expression for p_3 derived earlier from Boyle's law. We have derived the combined gas law. We see that the product $\frac{pV}{T}$ is the same in initial and final state. Analogously we could show that it stays the same in any two states of the gas. So, it is a constant in any process for a given gas:

$$\frac{pV}{T} = const.$$

This law contains all three gas laws what we previously discussed. Note that we only used two laws (Boyle's law and Charles's law) in derivation and as a result we see the third one

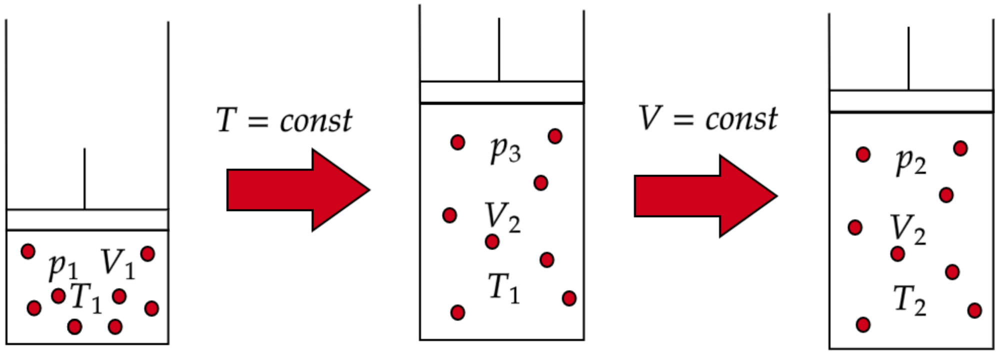

FIGURE 2. We could reach the final state p_2, V_2, T_2 in two steps : first keep temperature constant and then keep volume constant.

(Gay-Lussac's law) as well. So three gas laws were not really independent - we could just take two of them and the third would follow.

The value of combined gas law is that if we know all three parameters in a state 1 of the gas and we know two parameters in a state 2, we could find the remaining parameter in state 2.

HOMEWORK

1. Gas was heated from 27°C to 39°C but its pressure was maintained the same. By what percentage did the volume of the gas increase?

2. Temperature of a gas is increased 2 times (measured in Kelvin) and its' volume is decreased 3 times. How does the pressure change?

3. We have vertical cylinder with a piston of area A. The cylinder is filled with gas, which occupies volume V under the piston. The piston has mass m and can move without friction. What will happen to the gas volume if we move the cylinder vertically with acceleration a? Assume that you know the atmospheric pressure P_0 and gas temperature is kept constant.

***4.** A balloon probe is filled with gas at temperature $t_1 = 27°\text{C}$ up to pressure 105 kPa. After rising to height where the outside pressure is $p_0 = 80$ kPa the volume of the balloon increased by $n = 5\%$. The pressure in the balloon is higher than outside by $\Delta p = 5$ kPa. Find the temperature of outside air at this height assuming the gas inside the balloon has the same temperature as the outside air.