EQUATION OF STATE OF IDEAL GAS.
APRIL 3, 2022

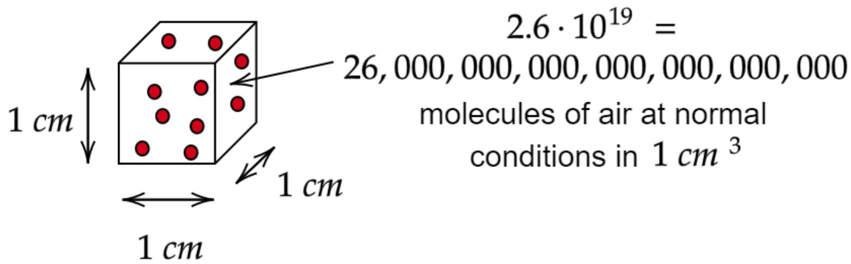
Theory recap

Last time we discussed combined gas law:

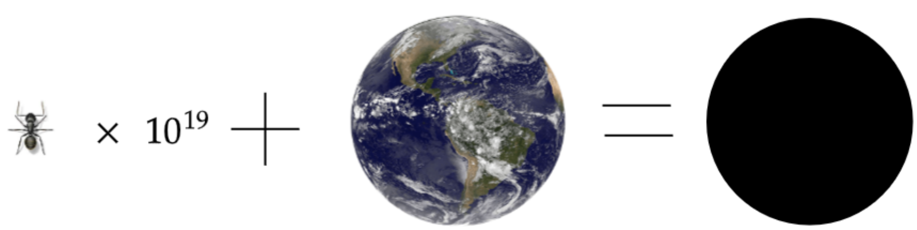
$$\frac{pV}{T} = const.$$

But what is the constant in the combined gas law equal to? It should be proportional to the amount of molecules of the gas. Why? Imagine that we consider two containers with gas, each having the same pressure and temperature but one being two times bigger in volume. Then in each half of the bigger container we should have the same amount of molecules as in the smaller container. Overall we have twice as many molecules in the big container then. $\frac{pV}{T}$ in the large container is also two times larger than in the small container (volume is two times bigger), so we see the relation between the constant in the combined gas law and the number of molecules.

Amount of molecules in gas. This brings us to the question: how many molecules are there in a gas? This of course depends on volume, as well as pressure and temperature. Let us take air at normal conditions - atmospheric pressure $p = 10^5$ Pa and temperature $T = 273$K (which is the same as 0° C) and consider a little cube with side 1 cm. The volume of this cube is 1 cm^3 and it has approximately $2.6 \cdot 10^{19}$ air molecules. How much is $2.6 \cdot 10^{19}$? If we write it out, it is 26,000,000,000,000,000,000 but it is still hard to appreciate how big this number is.



Let me put it this way: imagine an ant and the planet Earth. What if we tried to cover the entire surface of Earth (including the oceans) completely with a single layer of ants? How many ants would we need? This amount is actually of the same order of 10^{19} as the number of molecules in a single 1 cm^3 of air!



Amount of substance. Because there are so many molecules in ordinary objects around us it would not be very convenient to use the actual number of molecules in all computations. So people came up with a convention: for every substance we say that we have 1 mole of this substance if we have $6.02 \cdot 10^{23}$ molecules of this substance. This number is called the Avogadro number or the Avogadro constant:

$$N_A = 6.02 \cdot 10^{23}$$

If we have twice as many molecules we have two moles, and so on. The physical quantity measured in moles is called **amount of substance** and denoted by letter n. Mathematically it is defined like this:

$$n = \frac{N}{N_A} \text{ moles,}$$

where N is number of molecules in the substance and N_A is the Avogadro number.

Equation of state of ideal gas. Summing up, combined gas law is:

$$\frac{pV}{T} = nR$$

n is the amount of substance in moles. Everything in this formula is already familiar to us except R. R is called gas constant (or universal gas constant) and it is equal to

$$R = 8.31 \ \frac{\text{J}}{\text{mole} \cdot \text{K}}$$

R is another fundamental physical constant which we meet in our course. It describes the behavior of any gas, no matter its chemical composition.

Usually the last equation is written in the following equivalent form, which is a bit easier to remember because there are no fractions:

$$pV = nRT$$

This equation is very important. It is called the **equation of state of ideal gas**. It contains combined gas law and through it the three individual gas laws that we discussed, but actually is contains more than that. Equation of state of ideal gas tells us that all gases, no matter what atoms or molecules they consist of, have the same relation between four basic parameters: pressure, volume, temperature and amount of substance. So if you take the same amount of molecules of hydrogen or water vapor or chlorine, you wound not be able to distinguish them by the pressure they produce at given volume and temperature. If they have the same volume and temperature, they will have the same pressure. This is the great universality which is only found among gases because of their physical simplicity: in an ideal gas approximation they are just a bunch of molecules flying around without interacting with each other.

As an example of this universality let us calculate the volume that 1 mole of any gas would take at normal conditions: normal atmospheric pressure $p_0 = 101.3$ kPa and temperature $T = 0^\circ$ C $= 273$ K. From the equation of state of ideal gas we get

$$V = \frac{nRT}{p_0} = \frac{1 \cdot 8.31 \cdot 273}{101,300} \text{m}^3 = 0.0224 \text{ m}^3 = 22.4 \text{ L}$$

This number is useful to keep in mind, just to understand the scale of how much is one mole.

Equation of state of ideal gas also works for mixture of gases because it is only the total number of molecules that matters. If we take a mixture of 2 moles of oxygen with 3 moles of nitrogen, we will have equation of state for 5 moles of gas.

Molar mass. But how do we know the amount of substance of some matter? We could not count the number of molecules, of course. What we can easily do is measure the mass of substance. Could we maybe find number of moles from the mass? Yes! Each molecule of a substance has a particular mass, so $6 \cdot 10^{23}$ of these molecules also have a particular mass which is called molar mass and denoted by M. Now, if we have m kilograms of substance with mass M, the amount of substance is

$$n = \frac{m}{M}.$$

Equation of state of ideal gas could be written in terms of mass and molar mass:

$$pV = \frac{m}{M}RT$$

Homework

1. What is the volume of 3 moles of an ideal gas at the temperature of 57°C and pressure 150 kPa?

2. There is a 1 liter bottle filled with water at 27°C. The water is liquid at this temperature because there is attracting force between the molecules. Imagine that we have suddenly "turned off" this attracting force. What is the pressure in the bottle now? *Hint: mass of 1 liter of water is 1 kg, molar mass of water is 18 grams/mole.*

***3.** Gas is in a vessel at a pressure 2 MPa and temperature 27° C. After its' temperature is increased by 50° C, half (by mass) of the gas escapes the vessel. What is pressure in the new equilibrium state?