

IDEAL GAS PROCESSES. GRAPHICAL REPRESENTATION.

APRIL 10, 2022

THEORY RECAP

Last time recap. Last time we discussed ideal gas equation of state:

$$pV = nRT.$$

We also learned how to find the number of moles knowing mass m and molar mass M :

$$n = \frac{m}{M}$$

Molar mass from periodic table. How could we find molar mass of some substance? As we have seen, molar mass is related to mass of molecules of the substance. Mass of a molecule is equal to sum of masses of atoms comprising this molecule. And masses of atoms can be found in the periodic table of elements, which contains a lot of useful information about all the atoms.

Periodic Table of the Elements

1 IA 1A	2 IIA 2A											13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A						
1 H Hydrogen 1.008												5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180						
3 Li Lithium 6.941	4 Be Beryllium 9.012											11 Na Sodium 22.990	12 Mg Magnesium 24.305					13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.933	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.972	35 Br Bromine 79.904	36 Kr Krypton 84.00						
37 Rb Rubidium 84.455	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.905	54 Xe Xenon 131.29						
55 Cs Cesium 132.905	56 Ba Barium 137.327	57-71 Lanthanide Series	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.85	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium 209	85 At Astatine 209	86 Rn Radon 222						
87 Fr Francium 223	88 Ra Radium 226	89-103 Actinide Series	104 Rf Rutherfordium 261	105 Db Dubnium 262	106 Sg Seaborgium 266	107 Bh Bohrium 264	108 Hs Hassium 269	109 Mt Meitnerium 268	110 Ds Darmstadtium 269	111 Rg Roentgenium 272	112 Cn Copernicium 277	113 Uut Ununtrium unknown	114 Fl Flerovium 289	115 Uup Ununpentium unknown	116 Lv Livermorium 293	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown						
		57 La Lanthanum 138.905	58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.966	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.50	67 Ho Holmium 164.930	68 Er Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967							
		89 Ac Actinium 227	90 Th Thorium 232	91 Pa Protactinium 231	92 U Uranium 238	93 Np Neptunium 237	94 Pu Plutonium 244	95 Am Americium 243	96 Cm Curium 247	97 Bk Berkelium 247	98 Cf Californium 251	99 Es Einsteinium 254	100 Fm Fermium 257	101 Md Mendelevium 258	102 No Nobelium 259	103 Lr Lawrencium 262							

Legend: Alkali Metal, Alkaline Earth, Transition Metal, Basic Metal, Semimetal, Nonmetal, Halogen, Noble Gas, Lanthanide, Actinide

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There is a simple algorithm of finding molar mass from periodic table. First of all, we need to locate atomic mass in the periodic table: it is the lowest number in each cell. For example, for the first element - hydrogen (H) we can see the atomic mass is 1.008 which could be rounded to 1. This is exactly the molar mass of a hydrogen atom, measured in gram/mole. So, if we take 1 mole of hydrogen atoms, or $6 \cdot 10^{23}$ hydrogen atoms, their mass will be $M(\text{H}) = 1$ g/mole. If we take 1 mole of carbon atoms, their mass will be $M(\text{C}) = 12$ g/mole (find carbon C in the table above and verify that its' atomic mass is about 12).

Now, if we talk about molecules, molar mass is sum of molar masses of atoms building the molecules. For instance, nitrogen molecule is N_2 which means it consists of two nitrogen atoms. Therefore, one mole of nitrogen molecules has mass equal to two molar masses of nitrogen atoms:

$$M(N_2) = 2 \cdot M(N) = 2 \cdot 14 \text{ g/mole} = 28 \text{ g/mole}.$$

Let us do one more example. Consider a carbon dioxide molecule CO_2 which consists of a carbon atom and two oxygen atoms. From the periodic table we find that the molar mass of carbon atom C is 12 g/mole and that the molar mass of oxygen atom O is 16 g/mole. So we find molar mass of water:

$$M(CO_2) = M(C) + 2 \cdot M(O) = 12 + 2 \cdot 16 \text{ g/mole} = 44 \text{ g/mole}.$$

Processes with ideal gas. Our ultimate goal is to understand how gases could be used in machines to extract work from heat. For that we need to learn a bit about processes which could happen to gases and how to describe them conveniently. We have already understood that a gas in a given state is characterized by its pressure, volume and temperature and amount of moles. Assuming that we fix amount of moles and don't change it, we only need to know two parameters, for example pressure and volume, to specify the state of the gas. Temperature then can be found using the ideal gas equation of state.

Graphically we can represent state of the gas as a point in $(p - V)$ coordinate plane: every point corresponds to some particular values of pressure and volume. For example, let us take one mole of some gas with pressure $p_0 = 101,339 \text{ Pa}$ and volume $V_0 = 0.0224 \text{ m}^3$. This state is represented as a point on figure 1.

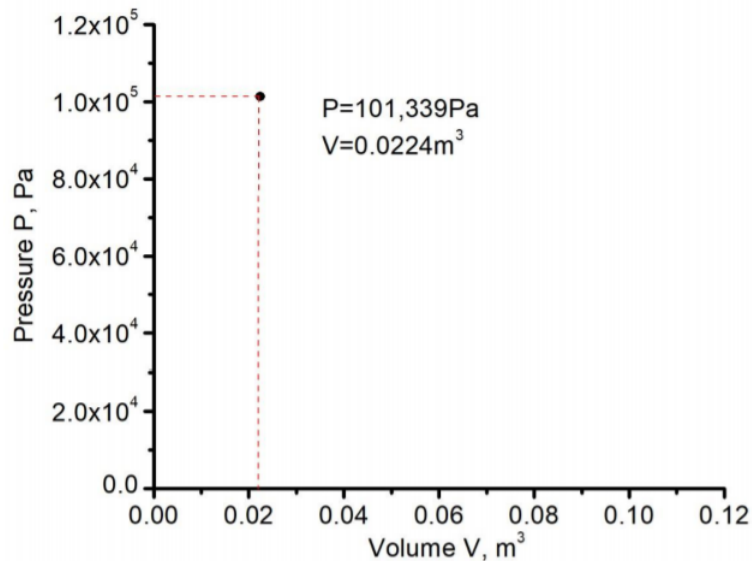


FIGURE 1. The point in $p - V$ coordinates represents the state in which the gas has pressure 101,339 Pa and volume 0.0224 m³.

Knowing pressure and volume we could find temperature in this state:

$$p_0V_0 = nRT_0 \implies T_0 = \frac{p_0V_0}{nR} = 273.16\text{K}$$

Now let us decrease pressure of the gas while keeping the volume constant (processes at constant volume are called **isochoric**). In this process the gas will go through many intermediate states, all with the same volume. On our plot it will be represented by a continuous line with every point on it corresponding to some intermediate state. Constant volume means the volume coordinate is fixed, so this should be a vertical line. Its' endpoint will be at the final pressure which we will take to be $p_1 = 20,000$ Pa.

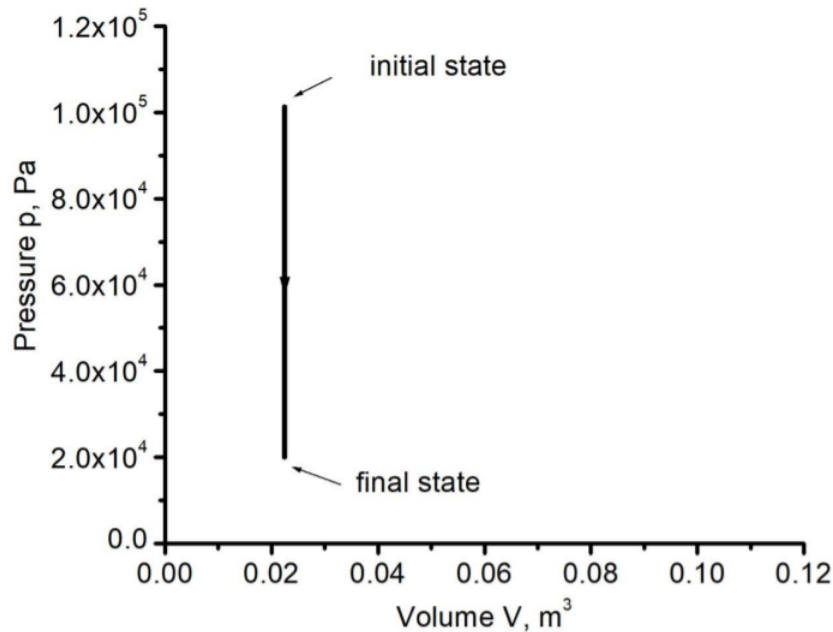


FIGURE 2. A vertical line in $p-V$ coordinates represents a process at constant volume, also called an isochoric process.

We could find the temperature T_1 in the final state with pressure p_1 by using either equation of state of ideal gas as above, or Gay-Lussac's law, which gives us

$$\frac{p_1}{T_1} = \frac{p_0}{T_0} \implies T_1 = T_0 \frac{p_1}{p_0} = 54 \text{ K.}$$

There is a possible caveat here, that 54 K is actually a really low temperature at which many gases, for example nitrogen or oxygen become liquid and therefore can not be described by ideal gas equation of state. But there are gases which only condense at much lower temperature, such as helium (at 4.2 K). So let us assume that here we work with helium and it behaves like an ideal gas at 54 K.

Now let us continue with our process. The next part will be done at constant pressure (processes at constant pressure are called **isobaric**) and increasing volume. Let us take the final volume to be $V_2 = 0.113 \text{ m}^3$. Process at constant pressure is represented by a horizontal line on our plot.

We could calculate temperature in the new final state this time using Charle's law:

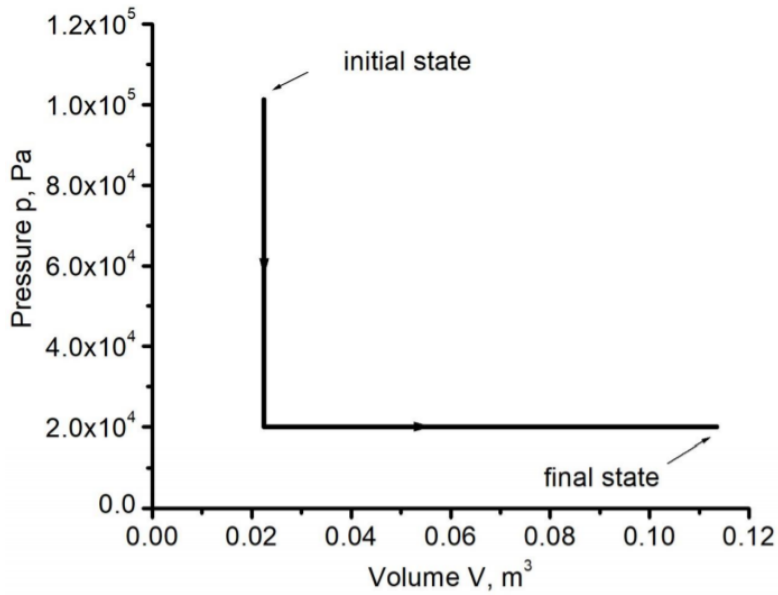


FIGURE 3. A horizontal line in $p - V$ coordinates represents a process at constant pressure, also called an isobaric process.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \implies T_2 = T_1 \frac{V_2}{V_1} = 273.16 \text{ K.}$$

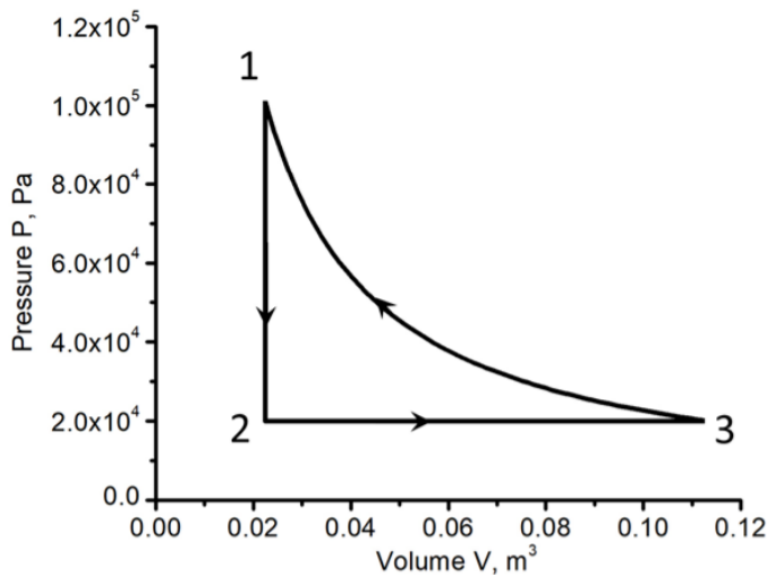


FIGURE 4. A hyperbola 3-1 in $p - V$ coordinates represents a process at constant temperature, also called an isothermal process.

So we have reached the same temperature as we had initially. We would like to return to the initial state because in order for us to be able to repeat the process again and again it should

be cyclic. The simplest opportunity now is to compress the gas at constant temperature, or **isothermally**. As we discussed some time ago, on the $p - V$ plot the corresponding curve is hyperbola, as shown on figure 4. The equation of this curve could be found from equation of state of ideal gas:

$$pV = nRT \implies p = \frac{nRT}{V} = \frac{2270 \text{ J}}{V}.$$

HOMEWORK

1. Find molar mass of molecular oxygen O_2 using periodic table. Using it, find the mass of oxygen in a 10 liter cylinder if it has temperature $T=13^\circ\text{C}$ and pressure $P = 9 \cdot 10^6$ Pa (note that it is 90x the normal atmospheric pressure!). For how long can the oxygen in this cylinder sustain a scuba diver, if an average person needs to inhale about 2 grams of oxygen per minute?
2. Consider the following cyclic process performed with 3 moles of ideal gas. We start from pressure 10 kPa and volume 2 m^3 (point 1). Then we isobarically (which means keeping constant pressure) compress the gas until volume reaches 0.5 m^3 (point 2). Then at constant volume pressure is increased up to 40 kPa (point 3). After that keeping the pressure constant we bring the volume up to the initial value 2 m^3 (point 4). Finally pressure is isochorically (which means keeping constant volume) reduced and the gas comes back to point 1. Draw a diagram of this process in $p - V$ coordinates and find the temperature of the gas at points 1,2,3 and 4.
- *3. Draw the cyclic process shown in Figure 4 in the coordinates p, T and V, T .