THE FIRST LAW OF THERMODYNAMICS.

MAY 1, 2022

Theory recap

Last time recap. Last time we discussed work done by gas and its' graphical representation. We have learned that in a process where pressure p of the gas is constant and its volume increases by ΔV, the work W done by the gas is calculated as follows:

$$W = p\Delta V.$$

For cyclic processes total work done by the cycle is numerically equal to the area enclosed inside the cycle depicted on a $p-V$ diagram.

Quantity of heat. Imagine heating a gas by a burner. The burner supplies some thermal energy to the gas. This energy coming from the burner is called quantity of heat and denoted by Q. Like all other kinds of energy, quantity of heat is measured in Joules.

First law of thermodynamics. Where does this energy go to? There are two possible ways. In order to identify them, let's first imagine that the temperature of the gas stays constant. Temperature of the gas is the measure of its internal energy and as temperature does not change, internal energy stays constant. When the volume increases, gas does work because of the heat supplied. Energy is always conserved, and since internal energy does not accumulate in the system, work W is just equal to the supplied amount of heat in this process:

$$W = Q \quad \text{for} \quad T = const.$$

On the other hand, let us look what happens if we fix the position of a piston. Then gas does not do any work and clearly because of the burner its temperature increases. Back when discussing temperature we've learned that temperature is a measure of internal energy of a substance. When temperature increases, internal energy also grows. Let us denote the change in internal energy by ΔE. Since the volume is fixed, no work is done by the gas. Therefore due to energy conservation all of the supplied heat Q should be accumulated in the system in the form of the internal energy:

$$\Delta E = Q \quad \text{for} \quad V = const.$$

What happens if we combine these two cases? Imagine we allow gas to change both its volume and temperature, which results in increase in internal energy of the gas by ΔE and work W done by the gas. The energy balance in this case works as follows: energy Q comes into the system from the outside, but energy W is spent on doing the work. Therefore the accumulation of internal energy is the difference between the supplied and spent energy:

$$\Delta E = Q - W.$$

This is **the first law of thermodynamics**. Essentially, it is nothing more but energy conservation law. Another common form of writing the first law of thermodynamics is as follows:

$$Q = \Delta E + W,$$

which means that all the heat supplied to the system could go either into work done by the system or into the change of internal energy.

Internal energy of the gas. For an ideal gas there is a simple relation between internal energy and temperature. In order to derive it, we will need two things. First, remember that temperature in Kelvins T is related to average kinetic energy of molecules of a substance as follows:

$$< E_{kin} >= \frac{3}{2}kT$$

where $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant. Second, remember that in an ideal gas there is no interaction between the atoms, so internal energy only consists of internal kinetic energy. If we have n moles of gas, there are $N = nN_A$ molecules of the gas and the overall kinetic energy is

$$E =< E_{kin} > \cdot N = \frac{3}{2}kT \cdot nN_A = \frac{3}{2}n(k \cdot N_A)T.$$

What is the product kN_A equal to? Remember that $k = 1.38 \cdot 10^{-23}$ J/K and $N_A = 6 \cdot 10^{23}$ $\frac{1}{\text{mole}}$. If we multiply these two numbers, we get $8.31 \frac{\text{J}}{\text{K}\cdot\text{mole}}$ - which we also encountered in our course as R, the universal gas constant! So internal energy of the gas has a very nice and simple expression in terms of temperature, number of moles and the universal gas constant:

$$E = \frac{3}{2}nRT.$$

Let me make a warning that there is a caveat in this derivation because of which it only works for very simple gas molecules which consist of only one atom (called monatomic gas). An example of such gas is helium, He. The caveat is that molecules which consist of several atoms perform more complicated motion including rotation or oscillations, which we haven't accounted for in our derivation when looking at the kinetic energy. It is not very hard to account for, but beyond the scope of our course.

Homework

1. 2 moles of helium are kept at constant volume and heated by a burner. How much does helium temperature increase if the burner supplies heat 500 J?

2. While 5 moles of helium were heated by an alcohol burner, helium's temperature increased by 300° C and helium performed 8000 J of work. Every gram of burned alcohol releases 26,000 J of heat. Assume that due to heat losses only 30% of heat released by the burner actually reaches the helium. What mass of alcohol was used to fuel the burner during this process?

***3.** 1 mole of monatomic gas is heated at constant pressure. Find what quantity of heat should be supplied in order to increase the temperature of the gas by 1° C. *Hint: use the expression for work done by gas and equation of state of ideal gas.*