

MATH CLUB
MATH BATTLE!!

DEC 19, 2021

1. Maria has a balance scale that can indicate which of its pans is heavier or whether they have equal weight. She also has 4 weights that look the same but have masses of 1001, 1002, 1004 and 1005 g. Can Maria determine the mass of each weight in 4 weighings? The weights for a new weighing may be picked when the result of the previous ones is known.
2. Can you divide a given triangle into 4 smaller triangles, so that any two of them are adjacent to each other? [Note: two triangles are called adjacent if they share an entire edge or a part of an edge.]
3. Is it true that if $A^3 + B^3 + C^3$ is divisible by 7, then ABC is divisible by 7?
4. A plot of land has the the shape of a 10×20 rectangle. It is known that this plot is crossed by a buried water pipe, going in a straight line. A plumber wants to find the pipe, by digging narrow trenches until one of them hits the pipe.

What is the shortest total length of trenches he has to dig to guarantee that he will find the pipe? One option is digging the trenches along two diagonals of the square. Can you do better than that?