WEIGHINGS AND GUESSING

FEB 6, 2022

1. How many questions you need to guess a number between 1-100 using yes/no questions? What about number between 1–1000?

2. You need to design a set of weights that would allow measuring any integer weight from 1 to 15 grams, using the balance scales (the item to be weighted is placed on one side; the weights, on the other). What is the smallest set of weights you can come up with? What if you need to measure any integer weight from 1 to 1000 grams?

3. You need to design a set of weights that woudl allow measuring any integer weight from 1 to 15 grams, using the balance scales (the item to be weighted is placed on one side; the weights can be placed on both sides). What is the smallest set of weights you can come up with? What if you need to measure any integer weight from 1 to 1000 grams?

4. A company needs to move 36 tons of products, packed in crates; each crate weighs at most 1 ton. They have a single truck that can move at most 4 tons.
   Is it true that they can always move the products making at most 11 trips? at most 10 trips?

5. A company is testing a new model of cellphones to find the maximal height from which it can be dropped without breaking. To do that, they built two identical prototypes and a 100 step high ladder (each step is 1 foot). How many tries do you need to determine the highest step from which the phone can be dropped without breaking?

6. You have 103 coins which look identical. However, two of the coins are counterfeit. It is known that all authentic coins have the same weight, and both counterfeit coins aslo weigh the same (but different from the real ones).
   Can you determine whether the counterfeit coins are lighter or heavier than the real ones in 3 weighings on a balance scales? You are not required to find the counterfeit coins.
   (Balance scales has two sides and show which side is heavier, but do not show what the weight is.)

7. You have 1000 blood samples; it is known that one of them contains a dangerous virus. You have a test that can test for a virus in any sample, but it takes 1 day to complete, and the lab can only run 10 such tests simultaneously (this is limited by the equipment they have). However, the test is so sensitive that he can detect the virus in very small concentrations — for example, if you mix several blood samples together, and one of them contains the virus, then running the test on the mix will show the virus. The test only requires very small amount of material, so each blood sample contains enough for many tests.
   Using this, how quickly can you find the infected sample?

*8. (a) A king has 9 barrels of wine; he suspects that one of them is poisoned and needs to find which one. He decides to test the wine on lab mice: it is known that if you give the mice any amount of poisoned wine, it will dy by next morning.
   How many days does he need to determine which barrel is poisoned if he only has two mice? [It is known that only one barrel is poisoned.]
   (b) Same question, but with 27 barrels and 3 mice.

9. A detective is working on a murder case. There are 80 people who were present in the area at the time of the murder; one of them is the murderer and one was a witness of the murder. To speed things up, the detective is doing group meetigns, inviting to every meeting some of these 80 people. If the witness is in this group and the murderer is not, the witness tells all to the detective. (If the murderer is there, the witness is too scared to talk.).
   Can the detective solve the case in 12 meetings? in 9 meetings?