

MATH CLUB
ASSIGNMENT 15: MATH BATTLE
FEB 27, 2022

1. Each of the quadratic polynomials $P(x)$, $Q(x)$ and $P(x) + Q(x)$ with real coefficients has a repeated root. Is it guaranteed that those roots coincide?
2. Mother baked 15 pastries. She placed them on a round plate in a circular way: 7 with cabbage, 7 with meat, and one with cherries in that exact order and put the plate into a microwave. All pastries look the same but Olga knows the order. However she doesn't know how the plate has been rotated in the microwave. She wants to eat a pastry with cherries. Can Olga eat her favourite pastry for sure if she is not allowed to try more than three other pastries?
3. There are 101 coins in a circle, each weights 10 g or 11 g. Prove that there exists a coin such that the total weight of the 50 coins to its left is equal to the total weight of the 50 coins to its right
4. Counters numbered 1 to 100 are arranged in order in a row. It costs \$1 to interchange two adjacent counters; it is free to interchange two counters with exactly 3 other counters between them. What is the minimum cost for rearranging the 100 counters in reverse order?
5. The director of a Zoo has bought eight elephants numbered by 1, 2, ..., 8. He has forgotten their weights but he remembers that each elephant starting with the third one has the weight equal to the sum of the weights of the two preceding ones. Suddenly the director hears a rumor that one of the elephants has lost weight. How to perform two weighings on balance scales without weights to either find this elephant or make sure that this was just a rumor? (It is known that no elephant gained weight and no more than one elephant lost weight.)