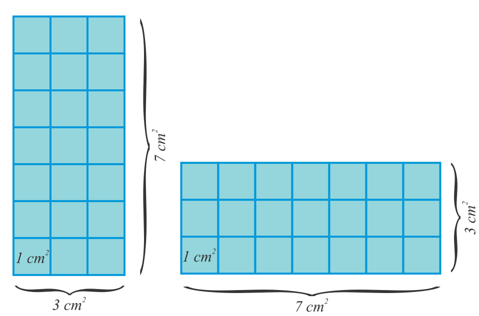
Math 4 d. Class work 2.

1. **Properties of the arithmetic operations.**

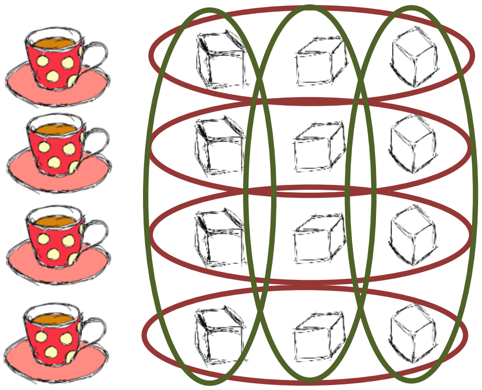
Addition		Multiplication
$a+b=b+a$	← Commutative property →	$a \times b=b \times a$
$a+(b+c)=(a+b)+c$	← Associative property →	$a \times (b \times c)=(a \times b) \times c$
	Distributive property ↓	
	$a \times (b+c)=a \times b + a \times c$ $a \times (b-c)=a \times b - a \times c$	

Commutative and associative properties of addition are easy to understand. Multiplication is just a shorter way to write the addition of equal groups, so commutative and associative properties of multiplication can be visualized and understood with the help of the rectangle area. (See the picture). Areas of identical rectangles are equal,



$$S = 3cm^2 \cdot 7 = 7cm^2 \cdot 3 = 3cm \cdot 7cm = 21cm^2$$

Let's take another look on the cumutative property of multiplication. How many cubes of sugar needed for four cups of tea, if three cubes are good for one cup?



There are two ways to calculate the number of sugar cubes:

- Put 3 cubes into each cup, $3\ cubes \cdot 4 = 12\ cubes$
- Or put one cube into each cup, then the second to the each cup, then the third.

$4\ cubes\ \cdot 3 = 12\ cubes.$

Of course, the results are the same, 12 cubes of sugar are needed for 4 cups of tea.

The distributive property can also be explained using the definition of multiplication:

$$2 \cdot (3+4) = (3+4) + (3+4) = 3+3+4+4 = 2 \cdot 3 + 2 \cdot 4$$

$$2 \cdot (3+4) = 2 \cdot 7 = 14\ and\ 2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$$

and it is true for any numbers.

We can do it he other way around:

$$2 \cdot 3 + 2 \cdot 7 = 3+3+7+7 = 3+7+3+7 = (3+7) + (3+7) = 2 \cdot (3+7)$$

The distributive property can be illustrated by the following problems:

Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether has farmer harvested if he has 10 boxes of green and 8 boxes of red grapes? Is there any difference between 2 following expressions?

$$5 \cdot (10+8) \text{ or } 5 \cdot 10 + 5 \cdot 8$$

What is represented by the first expression? By the second?

Another example:

For the party, John bought 7 identical boxes of chocolates, 20 chocolates each. The guests ate 12 candies from each box. How many candies are left after the party?

Again, two numerical expression can be written to describe the problem:

$$7 \cdot (20 - 12) \text{ and } 7 \cdot 20 - 7 \cdot 12.$$

For both examples we can write the equality:

$$7 \cdot (20 - 12) = 7 \cdot 20 - 7 \cdot 12$$
$$5 \cdot (10 + 8) = 5 \cdot 10 + 5 \cdot 8$$

These equalities are numerical representation of the distributive property, which can be written in the general form as

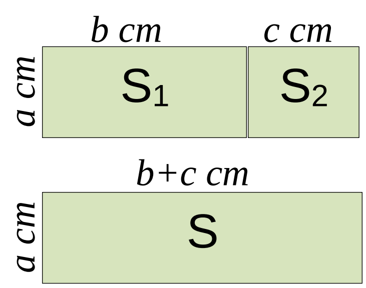
$$a \cdot (b + c) = a \cdot b + a \cdot c$$

and of cause

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

is also true, this way of writing the distributive property is called the factoring the common factor out (of the parenthesis).

The other way to see the distributive property is as a combined area of two rectangles with one side of the same length and the area of one rectangle. Combined area of two rectangles $S=S_1+S_2$ equals to



$a \cdot b + a \cdot c$, and the area of one big rectangle is

$S = a(b + c)$:

$$S = a(b + c) = a \cdot b + a \cdot c = S_1 + S_2$$

2. Factors.

Multiplication is a shorter way to write the addition of the same quantity:

$$\overset{\text{factor}}{5} \cdot \overset{\text{factor}}{3} = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = \overset{\text{product}}{15}$$

Numbers which are multiplied are called *factors*, the result of the operation is called *product*.

Also, we can tell that 15 can be factorized, represented as a product of 3 and 5.

3. Divisibility.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. It can be only if the divisor and the quotient are both factors of the dividend:

$a : b = c$, then $c \cdot b = a$.

c is a number, which gives a upon multiplication by b.

Take a look at some examples:

$$21 : 7 = 3 \rightarrow 3 \cdot 7 = 21; \quad 21 : 3 = 7 \rightarrow 7 \cdot 3 = 21$$

$$55 : 5 = 11 \rightarrow 11 \cdot 5 = 55; \quad 55 : 11 = 5 \rightarrow 5 \cdot 11 = 55$$

This relationship between dividend, divisor, and quotient can explain the reason of the fact, that the operation of division by 0 is not defined. We just can't divide by 0.

If we do, we will get a number that should be a factor of the initial number, but the result of multiplication by 0 is 0.

$5 : 0 = ?$; $100 : 0 = ?$ if there is such number, the product of this number by 0 should be 5, or 100 for the latter example, but it's not possible.

If there's no natural number that will give the dividend if multiplied by divisor, we can divide a number with a remainder.

$$\underbrace{a}_{\text{dividend}} : \underbrace{b}_{\text{divisor}} = \underbrace{c}_{\text{quotient}}$$

If a and n are natural numbers, the result of a division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$

$$\underbrace{a}_{\text{dividend}} = \underbrace{b}_{\text{divisor}} \cdot \underbrace{c}_{\text{quotient}} + \underbrace{r}_{\text{remainder}}$$

Where r is a remainder of a division $a \div b$. If r is 0, then we can tell that a is divisible by b.

- If we want to divide m by 15, what numbers we can get as a remainder?

If the remainder is 0, then quotient and divisor are both factors of dividend, $a = b \cdot c$, and the number a is divisible by b.

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

1. Evaluate the products and name the factors:
   Example: $3 \cdot 25 = 75, factors\ are\ 3\ and\ 25.$
   $a.\ \ 4 \cdot 12;$ $\quad b.\ 7 \cdot 11;$ $\quad c.\ 15 \cdot 20;$

2. Give several (two or more) examples of factorization (the representation of a number as product of two or more factors) for the numbers:
   Example: $120 = 1 \cdot 120 = 30 \cdot 4 = 15 \cdot 2 \cdot 4 = 15 \cdot 8;$
   $a.\ 50;$ $\quad b.\ 35;$ $\quad c.\ 49;$ $\quad d.\ 60,$ $\quad e.\ 48$

3. Using the distributive property fill the empty spaces:
   Example:
   $a.\ 5 \cdot (\square + 7) = 40 + \square$ $\qquad b.\ (\square - \square) \cdot 2 = 18 - 8$
   In your notebook:
   a. 5· (8+7) = 5·8+5·7=40+35 $\qquad$ b. (9-4)·2=9·2-4·2=18-8

   $c.\ \ 11 \cdot (2 + \square) = \square + 77;$ $\qquad d.\ \ \square \cdot (35 - 5) = 70 - \square$

   $e.\ \ \square + 10 = 2 \cdot (2 + \square);$ $\qquad f.\ \ 34 - \square = 17 \cdot (\square - 1)$

4. Prove that the number 425 is divisible by 17? (17 is a divisor of 425)
5. Write all divisors of numbers: 8, 12, 15, 36
   Example: $D(8) = \{1, 2, 4, 8\}$

6. Is the product of 1247 and 999 divisible by 3?

7. Number a is divisible by 5. Is the product $a \cdot b$ divisible by 5?

8. Without calculating, establish whether the product is divisible by a number?

   $a.\ 508 \cdot 12\ by\ 3$ $\qquad$ $b.\ 85 \cdot 3719\ by\ 5$

   $c.\ 2510 \cdot 74\ by\ 37$ $\qquad$ $d.\ 45 \cdot 26 \cdot 36\ by\ 15$

   $e.\ 210 \cdot 29\ by\ 3, by\ 29$ $\qquad$ $f.\ 3800 \cdot 44 \cdot 18\ by\ 11, 100, 9$

9. Show that the sum of two any even natural numbers is an even number.

10. Without calculating, establish whether the sum is divisible by a number:

    $a.\ 25 + 35 + 15 + 45 \quad by\ 5;$ $\qquad$ $b.\ 14 + 21 + 63 + 24\ by\ 7$

    $c.\ 18 + 36 + 55 + 90 \quad by\ 9;$

11. How many vans are needed to take 32 students on a field trip if a van can take 6 students? What is the maximal number of vans that can be fully occupied by these students?

12. The summer vacation is 73 days long. Which day of the week will be last day of vacations if the first day was Tuesday?

13. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating? Explain.

14. Find all natural numbers such that upon division by 7 the quotient and remainder will be equal.

15. Even or odd number will be the sum and the product of

    a. 2 odd numbers
    b. 2 even numbers
    c. 1 even and 1 odd number
    d. 1 odd and 1 even number

    Can you explain why? (a few examples do not prove the statement).

16. Show that among any three consecutive natural numbers there will be one divisible by 3.

17. Among four consecutive natural numbers will be a number

a. Divisible by 2?
b. Divisible by 3?
c. Divisible by 4?
d. Divisible by 5?

18. Compute (what is the best way to compute it? Hint: use the distributive property):

a. $23 \times 15 + 15 \times 77;$
b. $79 \times 21 - 69 \times 21;$
c. $340 \times 7 + 16 \times 70;$
d. $250 \times 61 - 25 \times 390;$
e. $67 \times 58 + 33 \times 58;$
$f.$ $55 \times 682 - 45 \times 682;$

19. You have a 3-gallon and a 5-gallon jug that you can fill from a fountain of water. How you can fill one of the jugs with *exactly* 4 gallons of water?

Geometry.

A **definition** is a statement of the meaning of a something (term, word, another statement).

desk
noun
noun: **desk**; plural noun: **desks**

1. a piece of furniture with a flat or sloped surface and typically with drawers, at which one can read, write, or do other work.

nonyms: iting table, bureau, escritoire, secretaire, rolltop desk, carrel, rkstation, worktable

- Music
  a position in an orchestra at which two players share a music stand.

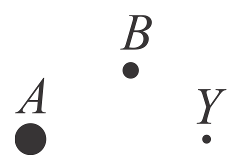
"an extra desk of first and second violins"

- a counter in a hotel, bank, or airport at which a customer may check in or obtain information.
  "the reception desk"

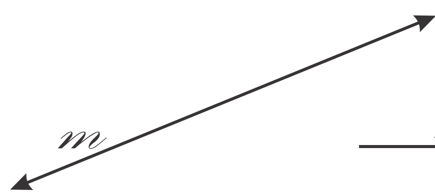
In mathematics everything (mmm,,,, almost everything) should be very well defined. In our real life it is also very useful and convenient to agree about terms and concepts, to give them a definition, before starting using them just to be sure that everybody knows what they are talking about.

Can we give a definition to a point? Can we clearly define what a point is? What a line is? What a plane is?

Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

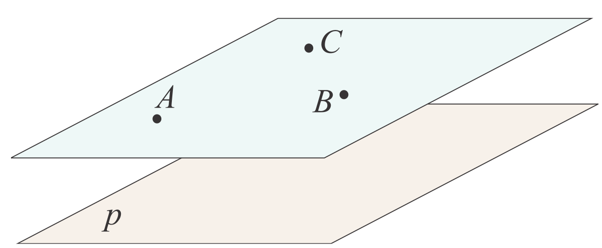


Point (an undefined term).
In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.



Line (an undefined term).
In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, *m* for example, or by any two points on the line, $\overleftrightarrow{(AB)}$ or (AB).

Plane (an undefined term).
In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that



the plane has no boundaries. A plane is named by a single letter (plane *p*) or by three non-collinear points (plane ABC).

A set of all points of a straight line between two points.
These points are called endpoints.

A ray is a part of a straight line consisting of a point (endpoint) and all points of a straight line at one side of an endpoint. Ray is named by endpoint and any other point, ray $\overrightarrow{AB}$ or *AB* (where *A* is an endpoint)

Exercises:

20. Draw two line segments [AB] and [CD] in such way that their intersect
    a. by a point
    b. by a segment
    c. don't intersect at all.
21. Using a ruler draw a straight line, mark 3 points on the line, *A*, *B*, and *C* so that 2 rays are formed, *BC* and *BA*.
22. Draw three lines so, that they produce
    a. One point of intersection
    b. Two points of intersection
    c. Three points of intersections
    d. No intersections

    (point of intersection is a point where two or more lines intersect)
23. Draw two rays $[\overrightarrow{AB})$ and $[\overrightarrow{CD})$ in such way that their intersect
    d. by a point
    e. by a segment
    f. by a ray
    g. don't intersect at all