Math 4d. Classwork 6.

About variables.

When we need to write a mathematical expression, but we don't know the exact numbers to use, we use variables. It can be any character, but it is very convenient to use letters. For example, if the number of books on the first shelf is n and the number of books on the second shelf is m, the total number of books on both shelves is $n + m$. We can do all the usual arithmetic operations on variables, but the exact answer can only be obtained when values are passed into variables.

Write the expression for the following problems:

- 3 packages of cookies cost a dollars. How many dollars do 5 of the same packages cost?

If 3 packages of cookies cost a dollars, one pack is cost

$$1pack = \frac{a}{3} = a:3$$

Five such packs will be

$$5 \cdot a:3 = \frac{5a}{3} = \frac{5}{3}a$$

- 5 bottles of juice cost b dollars. How many bottles can one buy with c dollars?

Again, if 5 bottles cost b dollars, one bottle will cost

$$\frac{b}{5} \text{ dollars}$$

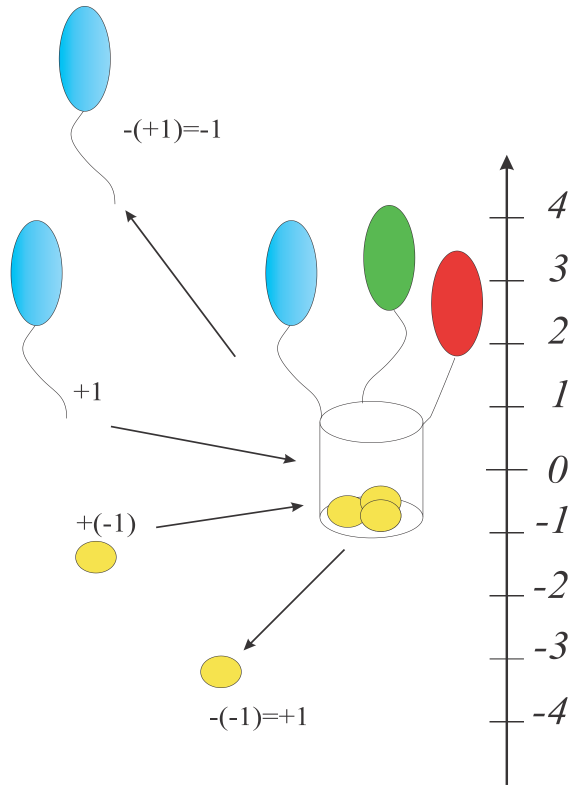
If I have only c dollars, I can buy the number of bottles equal to my total money divided by the price of one bottle:

$$c:\frac{b}{5} = c \cdot \frac{5}{b} = \frac{5c}{b}$$

If I have only \$30 and 5 bottles cost 10 dollars I can buy:

$$30:\frac{10}{5} = 30 \cdot \frac{5}{10} = 30 \cdot \frac{1}{2} = 15 \text{ bottles}$$

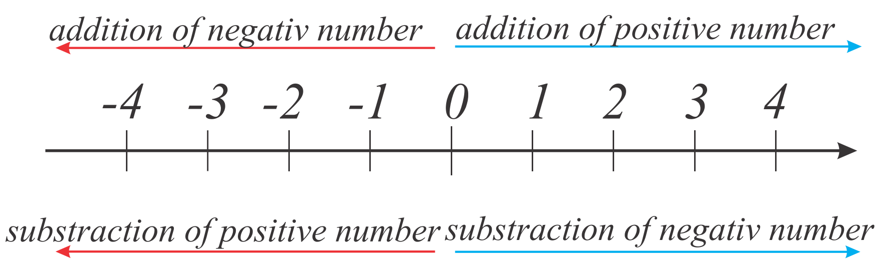
Positive and negative numbers.



Negative numbers represent opposites. If positive represents movement to the right, negative represents movement to the left. If positive represents above sea level, then negative represents below level. If positive represents a deposit, negative represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty (202 BC – AD 220), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as Diophantus considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd

Last year when we discuss the negative numbers we used very clear analogy of a basket with balloons and sand bags.

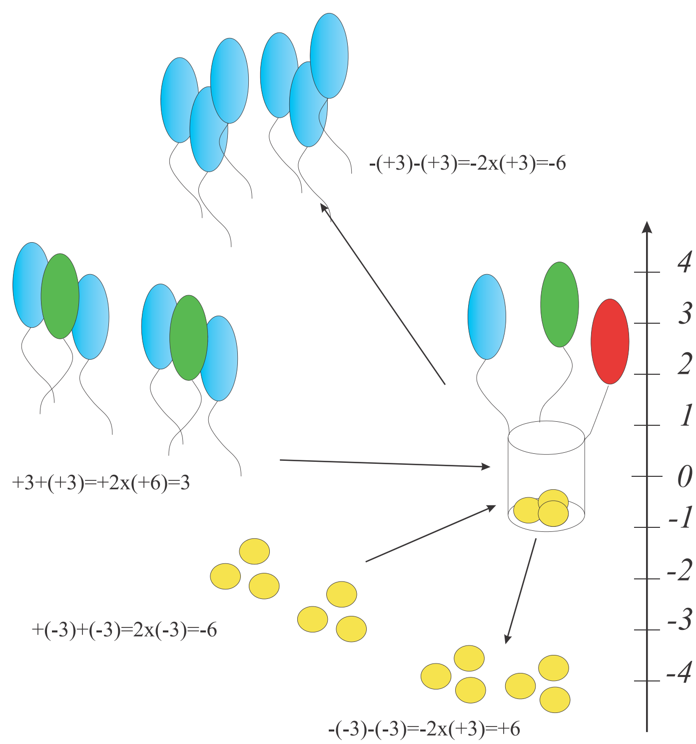
At the beginning basket has N balloons and N sand bags, placed at 0 position and doesn't move. Balloons represent positive units, sand bags represent negative units. If we add one balloon the basket will move one unit up. If we add one sand bag basket will move one unit down. If we remove one balloon, basket will go one unit down, which is equivalent of adding one sand bag. So $-(+1) = +(-1)$. If we remove one sand bag, basket will go one unit up, which is equivalent of adding one balloon. So $-(-1) = +(+1)$. Let's move to number line:



Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

Multiplication and division of negative numbers.

If we multiply 2 positive numbers, we will get third positive number. What will happened if we multiply one negative and one positive number. Let's again review our model. In this case we will add or remove our balloons and sand bags by groups of three. Addition of two groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So $2 \times (-3) = -6$. (We know that $-1 + (-1) + (-1) = -3$)



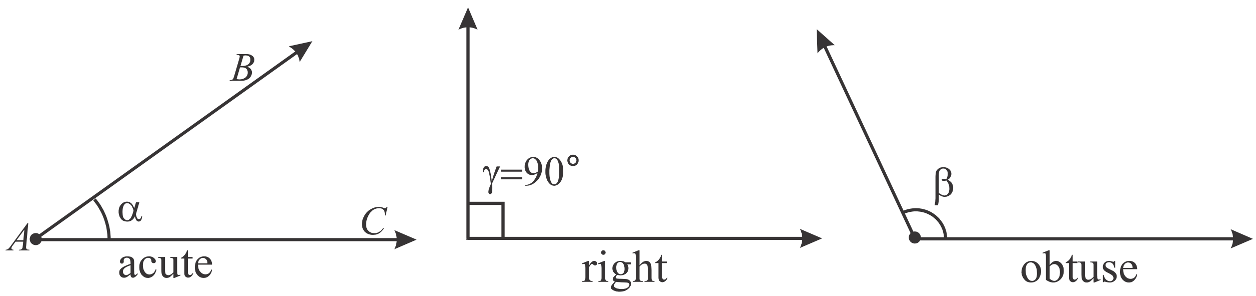
Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so $-2 \times (-3) = 6$

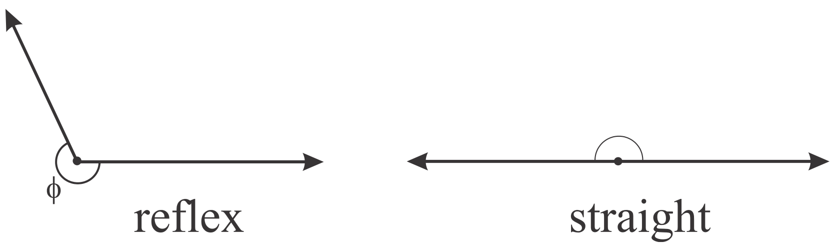
Addition of 6 balloons (2 groups of three) of cause will help us to move up for 6 units. If we remove 2 groups of 3 balloons we will descend 6 units. $-2 \times (+3) = -6$.

Geometry.

An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

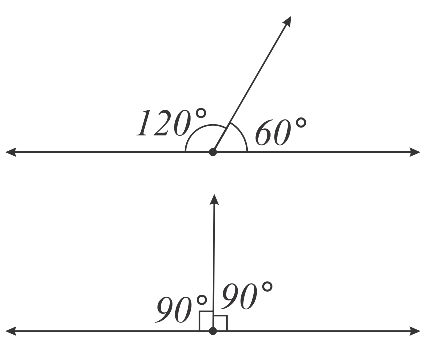
Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: ∠ABC, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:



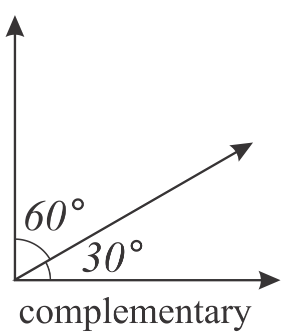


Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.



supplementary



An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

Exercises:

1. Apple costs x dollars and pear costs y dollars. Explain the expressions below:

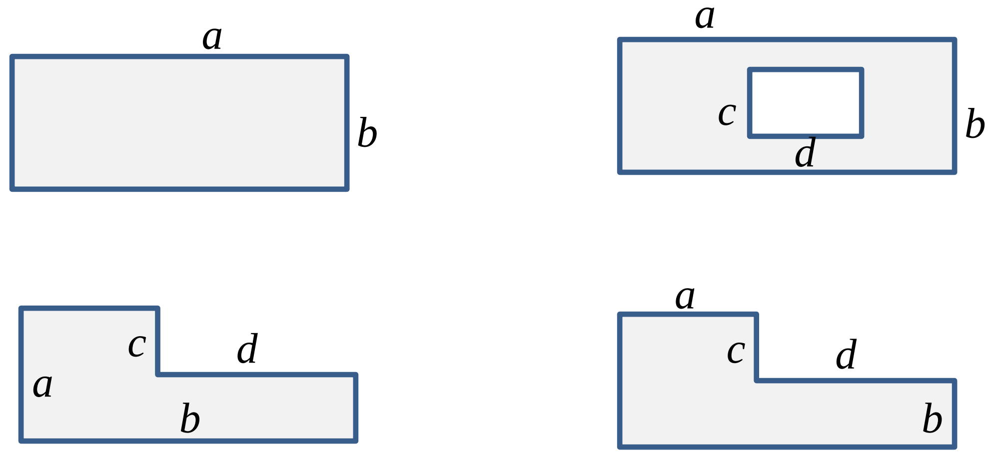
$$x + y, \qquad x - y, \qquad 3x, \qquad 8y, \qquad 3x + 8y, \qquad y : x, \qquad 120 : y$$

2. Write a single numerical expression to solve the problem:
There are 25 students in a class. After school, 7 students went home, and the rest made 3 equal teams to play basketball. How many students are in each team?

3. Alex is m years old. Robert is n years older than Alex. How many times Alex will be younger than Robert in 3 years? Solve the problem for $m = 2, n = 10$.

4. Julia had 20 cards. She gave a cards to her sister. How many cards she has now? Can a be any number?

5. Write the expressions for the shaded areas below (all angles are right angles):



6. Fill up the table:

a	7	-4			5		0	
$-a$			0	-1		8		-3

7. Rewrite without parenthesis:
Example:
$30 - (2 + 1) = 30 - 2 - 1, \quad 30 - (2 - 1) = 30 - 2 + 1$

To check your solution, you can find the value for both part of the equality:

$30 - (2 - 1) \neq 30 - 2 - 1$

$30 - (2 - 1) = 30 - 1 = 29; \quad 30 - 2 - 1 = 27.$

$30 - (2 - 1) = 30 - 2 + 1;$

$30 - (2 - 1) = 30 - 1 = 29, \quad 30 - 2 + 1 = 29$

$a. \quad 20 + (2 - 3);$ $\qquad b. \quad 20 - (2 - 3);$

$c. \quad 20 - (-2 + 3);$ $\qquad d. \quad 20 - (-2 + (-3));$

8. Compare:

$-4 \quad 4$	$6 \quad -4$	$\frac{2}{3} \quad -\frac{3}{2}$
$-4 \quad -2$	$-4 \quad 0$	$-\frac{2}{3} \quad -1$
$-4 \quad -6$	$-1 \quad -\frac{1}{2}$	$-2 \quad \frac{1}{2}$

9. Evaluate:

$3 + (-2);$	$3 + (+2);$	$-3 - (-2);$
$3 - (+2);$	$-3 + (-2);$	$-3 + (+2);$
$3 - (-2);$	$-3 - (+2);$	$-3 + (+3);$

10. Compare without calculation.

$a. \quad 100 - (35 - 20) \qquad 100 - (35 + 20)$

$b. \quad 100 + (35 - 20) \qquad 100 + (35 + 20)$

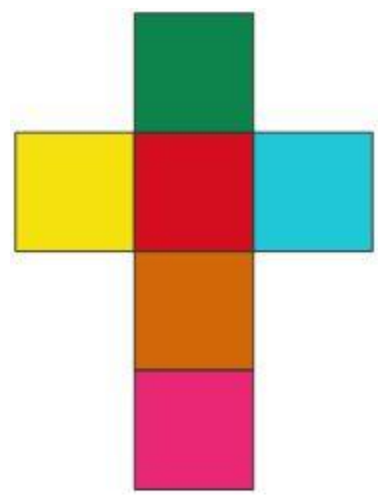
$c.\quad 100 - (-35-20) \quad 100-(-35+20)$

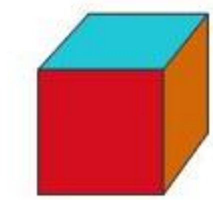
$d.\quad 100+(-35-20) \quad 100+(-35+20)$

11. Positive or negative number will be the product of
    a) Two negative and one positive numbers.
    b) One negative and two positive numbers
    c) Three negative numbers.

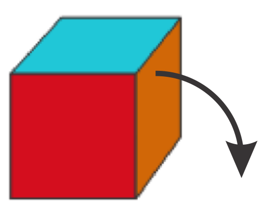
12. 25 identical thick books or 45 identical thin books can fit on a bookshelf. Will there be enough space on a bookshelf for 20 thick and 9 thin books?

13. On a picture on the right there is a surface of a cub. What do you think about the color of bottom side of this cub?

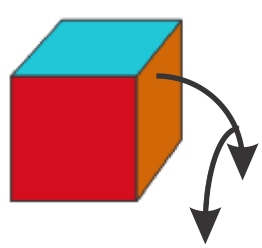




If you turn this cube ones following the arrow, what color of the upper side will be?



If you turn this cube one more time following the second arrow, what color of the upper side will be?



14. Fill the empty spaces in the table below:

a	56		36		72
b	8	6		5	
a · b		108	144		
a : b				14	24

15. Numbers 100 and 90 were divided by the same number. In the case of 100, the remainder is 4, in the case of 90, the remainder is 18. What is the divisor?