Math 4d. Classwork 9.

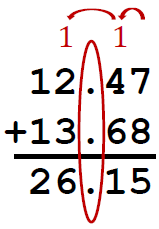
Algebra.

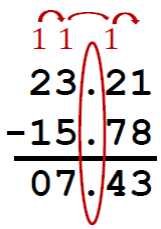
Today we are going to do some arthmetic operation with decimals.
First, let see how our normal addition and subtraction work when we do them with natural numbers and decimals.

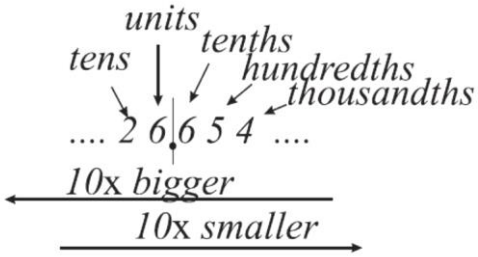
$$1247 + 1368 = (1 \cdot 1000 + 2 \cdot 100 + 4 \cdot 10 + 7) + (1 \cdot 1000 + 3 \cdot 100 + 6 \cdot 10 + 8)$$
$$= 2 \cdot 1000 + 5 \cdot 100 + 10 \cdot 10 + 15 = 2 \cdot 1000 + 5 \cdot 100 + 10 \cdot 10 + 10 + 5$$
$$= 2 \cdot 1000 + 5 \cdot 100 + 11 \cdot 10 + 5 = 2 \cdot 1000 + 5 \cdot 100 + 100 + 10 + 5$$
$$= 2 \cdot 1000 + 6 \cdot 100 + 1 \cdot 10 + 5 = 2615$$

In your notebooks do this addition in column.

This addition operation is very similar when used with decimal numbers; decimal notation is adopted to our place value number base 10 system (two consecutive place values are 10 times different).







To perform addition or subtraction with decimals using column method, both numbers should be written one under another in a way that decimal points are aligned, as shown on the pictures above.

Multiplication.

$$234 \cdot 10 = (200 + 30 + 4) \cdot 10 = 200 \cdot 10 + 30 \cdot 10 + 4 \cdot 10 = 2000 + 300 + 40 + 0$$
$$= 2340$$

Using the distributive property, we have just shown that when we need to multiply any natural number by 10, we just need to write 0 at the end of a number, increasing all place values 10 times. (Same goes for multiplication by 100 and so on….).

$$23.4 \cdot 10 = (20 + 3 + 0.1 \cdot 4) \cdot 10 = 20 \cdot 10 + 3 \cdot 10 + 0.1 \cdot 4 \cdot 10 = 200 + 30 + 4 = 234$$

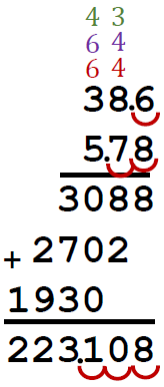
$$23.45 \cdot 10 = (20 + 3 + 0.1 \cdot 4 + 0.01 \cdot 5) \cdot 10 = 20 \cdot 10 + 3 \cdot 10 + 0.1 \cdot 4 \cdot 10 + 0.01 \cdot 5 \cdot 10$$
$$= 200 + 30 + 4 + 0.1 \cdot 5 = 234.5$$

These are two examples of how the numbers in decimal notation should be multiplying by 10. Using the distributive property, we proved that the result will be the number with decimal point moved one step to the right. (2 steps for multiplication by 100, and so on). Below are two examples of division by 10 (wich can be seen as multiplication by $\frac{1}{10}$)

$$230:10 = 230 \cdot \frac{1}{10} = (200 + 30 + 0) \cdot \frac{1}{10} = \frac{200}{10} + \frac{30}{10} + \frac{0}{10} = 20 + 3 = 23$$

$$235:10 = 235 \cdot \frac{1}{10} = (200 + 30 + 5) \cdot \frac{1}{10} = \frac{200}{10} + \frac{30}{10} + \frac{0}{10} = 20 + 3 + \frac{5}{10} = 23.5$$

To perform the long multiplication of the decimals, we do the multiplication procedure as we would do with natural numbers, regardless the position of decimal points, then the decimal point should be placed on the resulting line as many steps from the right side as the sum of decimal digits of both numbers. When we did the multiplication, we didn't take into the consideration the fact, that we are working with decimals, it is equivalent to the multiplication of each number by 10 or 100 or 1000 … (depends of how many decimal digits it has). So, the result we got is greater by $10 \cdot 100 = 1000$ time then the one we are looking for.



$$38.6 \cdot 5.78 = (38.6 \cdot 10 \cdot 5.78 \cdot 100):(10 \cdot 100) = (386 \cdot 578):1000$$

Division of a natural number by another natural number, not a divisor. We always can add any number of 0 after decimal point: 50=50.0000, 24.6=24.6000. The decimal point in the quotient is placed at the moment when the units in the dividend are divided (a): four goes to 5 one time, so 10 (1 and 0 from 50) need to be divided by 4, quotient of this division is 2 (and remainder is 2). Here the whole part of 50 ends, and decimal point should be

a)
$$\begin{array}{r} 12.5 \\ 4\overline{)50.0} \\ -4 \\ \hline 10 \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

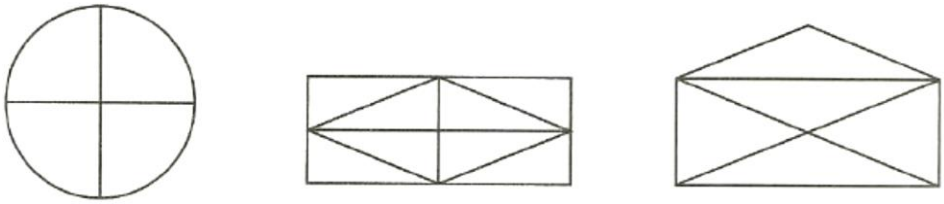
b)
$$\begin{array}{r} 4.92 \\ 5\overline{)24.6} \\ -0 \\ \hline 24 \\ -20 \\ \hline 46 \\ -45 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

c)
$$\begin{array}{r} 35.2 \\ 23\overline{)809.60} \\ -69 \\ \hline 119 \\ -115 \\ \hline 46 \\ -46 \\ \hline 0 \end{array}$$

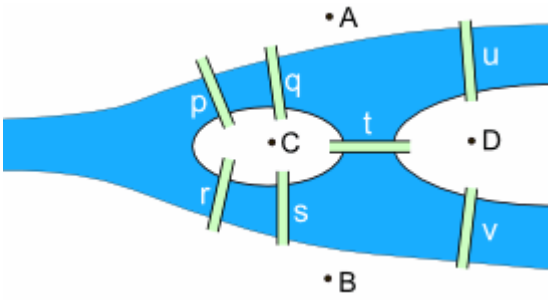
placed in the answer, also 0 should be dropped down to the remainder 2. Next step will give us the last digit in the answer, $20:4=5$. Division of the number with the fractional part, expressed in decimal notation should be done the same way. If we need to divide by a fractional number with decimal, we just need to multiply both dividend and divisor by 10 (100, 1000 …., depends of how many digits there are after decimal point and divide as described above).

Geometry.

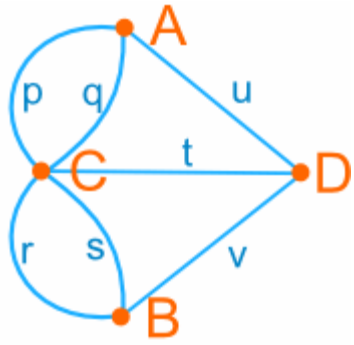
We did many problems about how to draw a picture without tracing twice any segment in a figure. Can you tell right away which figure can be traced this way and which cannot?



The old town of Königsberg has seven bridges:



Can you take a walk through the town, visiting each part of the town and crossing each bridge only once? We can mark each part of the town as A, B, C, and D and each bridge as **p, q, r, s, t, u,** and **v** and redraw map as below:

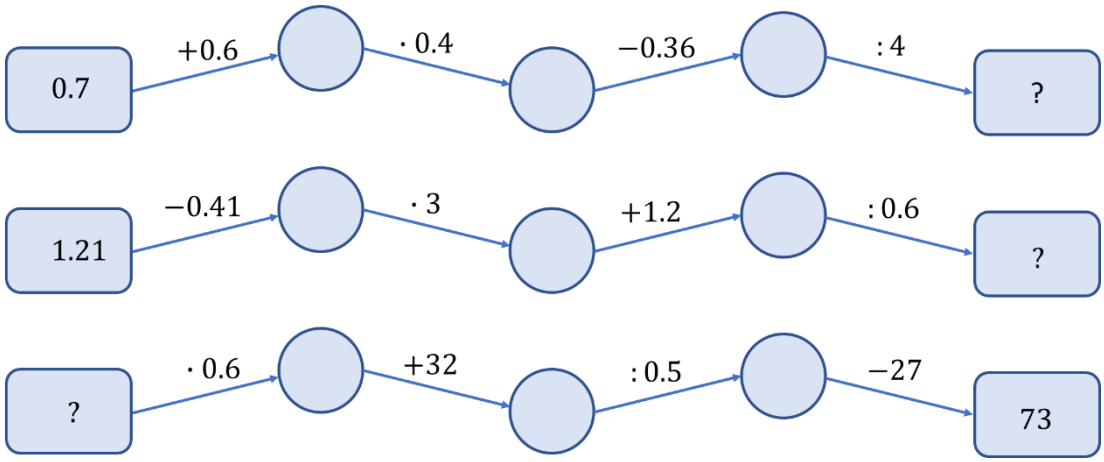


Try to pass by each line once and visit each point.

We created a graph. A point is a vertex of the graph and a line is called an edge. Vertex can be even or odd. If the number of edges going in (and out) is even, vertex is even vertex. Is all vertex of the graph are even, you can visit all vertex without passing twice any edge. If there are two odd vertices, we can start from one and end at the other. If a graph has more then two odd vertices, it can't be traced (without passing twice some edges)

Exercises:

1. What number should be placed instead of "?"



2. Evaluate:

   a. $6.57 + 23.345$; *b*. $45.67 - 23.34567$; *c*. $56 + 324.547$;
   d. $3.23 \cdot 0.7$; *e*. $5.23 \cdot 1.2$; *f*. $67.8 \cdot 21$;

3. Evaluate:

   a. $2.45 \cdot 10$;
   b. $0.54 \cdot 1000$;
   c. $1.25 \cdot 10000$;
   e. $9.674 \cdot 100$;
   f. $0.39 \cdot 10$;
   g. $34.9 \cdot 100$;

4. A few kids went to the forest to pick mushrooms. If Anya gives half of her mushrooms to Vita, all the children will have equal number of mushrooms, if instead Anya gives all her mushrooms to Sasha, then Sasha will have as many mushrooms as all the other kids combined. How many kids went to the forest for mushrooms?

5. Evaluate:

   a. $24.5 : 10$;
   b. $0.54 : 1000$;
   e. $967.4 : 100$;
   f. $0.39 : 10$;

c. $12556.7 : 10000$; *g*. $34.9 : 100$;

6. Compare, if possible:

$|7+3| \quad |7|+|3|$

$|7-3| \quad |3-7|$

$|a-b| \quad |b-a|$

$|3a| \quad 3 \cdot |a|$

$|7-3| \quad |7|-|3|$

$|3-7| \quad |3|-|7|$

$|7-3| \quad |7|+|3|$

$|a+b| \quad |a|+|b|$

$|b \cdot a| \quad b \cdot |a|$

7. Aunt Sally asked Tom Sawyer to paint $\frac{2}{5}$ of the whole fence. He asked his friend Ben Rogers to help him and Ben painted $\frac{1}{4}$ of that part of the fence. What is the length of the fence if Ben painted $2\frac{1}{2}$ m.

8. On a grid (graph) paper draw the coordinate system. Mark the points A(0;2), B(2;6), C(8;8), D(6,4). Draw the quadrilateral. Find the coordinate of the intersection of the diagonals. Use ruler! Try to be accurate!

9. ABCD is a rectangle. Find the coordinates of point D and draw the rectangle on a graph paper.

   a. A(-9; 2), B(-9; 4), C(-3; 4)
   b. A(0; 6), B(0; -2), C(5, -2)
   c. A(9; 0), B(9, -5), C(2, -5)
   d. A(-6; 0), B(-6;-7), C(0; -7)

10.