Math 4d. Classwork 15.

$$\frac{7^5}{7^3} = \frac{7\cdot 7\cdot 7\cdot 7\cdot 7}{7\cdot 7\cdot 7} = \frac{7\cdot 7\cdot \cancel{7}\cdot \cancel{7}\cdot \cancel{7}}{\cancel{7}\cdot \cancel{7}\cdot \cancel{7}} = 7\cdot 7 = 7^2$$

$$\frac{7^5}{7^3} = 7^2 = 7^{5-3} = 7^{5+(-3)}$$

1. $a^n = \underbrace{a\cdot a\cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n\cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a\cdot b)^n = a^n \cdot b^n$

We can see that when we multiply (same) numbers in power, the exponent indices ("indices" is a plural of "index") should be summed. If we divide such numbers, the power of the divisor should be subtracted from power od dividend.

$$\frac{7^3}{7^4} = \frac{7\cdot 7\cdot 7}{7\cdot 7\cdot 7\cdot 7} = \frac{1}{7} = 7^{3-4} = 7^{-1}; \qquad 7^{-1} = \frac{1}{7}$$

We can say that 7^{-1} is equal to 1/7, or to division of 1 by 7.

$$\frac{7^3}{7^6} = \frac{7\cdot 7\cdot 7}{7\cdot 7\cdot 7\cdot 7\cdot 7\cdot 7} = \frac{1}{7\cdot 7\cdot 7} = 7^{3-6} = 7^3\cdot 7^{-6} = 7^{-3} = \frac{1}{7^3}$$

Or, in a general way:

$$a^{-n} = \frac{1}{a^n} \qquad (a \neq 0)$$

Let's take a look on a few examples:

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9}$$

But also, we can write:

$$\left(\frac{2}{3}\right)^2 = \left(2\cdot\frac{1}{3}\right)^2 = 2^2\cdot\left(\frac{1}{3}\right)^2 = 2\cdot 2\cdot\frac{1}{3}\cdot\frac{1}{3} = 4\cdot\frac{1}{9} = \frac{4}{9}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}; \qquad \left(\frac{1}{3}\right)^{-1} = \frac{1}{\frac{1}{3}} = 1 \cdot 3 = 3;$$

$$\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \frac{1}{\frac{3}{4} \cdot \frac{3}{4}} = \frac{1}{\frac{9}{16}} = \frac{16}{9} = \left(\frac{4}{3}\right)^2$$

We can write any number in the extended form using the power of 10:

$$5478 = 12000 \cdot 5 + 100 \cdot 4 + 10 \cdot 7 + 1 \cdot 8 = 10^3 \cdot 5 + 10^2 \cdot 4 + 10^1 \cdot 7 + 10^0 \cdot 8$$

We can write any number, big and small (decimal part can be extended with a concept of negative power:

$$7865.234 = 1000 \cdot 7 + 100 \cdot 8 + 10 \cdot 6 + 1 \cdot 5 + \frac{1}{10} \cdot 2 + \frac{1}{100} \cdot 3 + \frac{1}{1000} \cdot 4$$
$$= 10^3 \cdot 7 + 10^2 \cdot 8 + 10^1 \cdot 6 + 10^0 \cdot 5 + 10^{-1} \cdot 2 + 10^{-2} \cdot 3 + 10^{-3} \cdot 4$$

n	1	2	3	4	5	6
$\mathbf{10^n}$	10	100	1000	10000	100000	1000000

n	1	2	3	4	5	6	7	8	9	101
$\mathbf{2^n}$	2	4	8	16	32	64	128	256	512	024

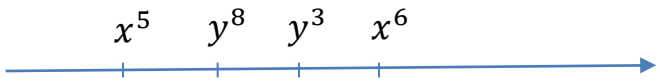
Exercises:

1. Write as a second power of a number following numbers:
   25, 121, 144, 225

2. Write as a third power of a number following numbers:
   1, 125, 512, 1000

3. The sequence of numbers contains 5 numbers, the first number is 10, after that each number is 10 times greater than the previous one. Write this sequence, representing all numbers to the power of 10.

4. $x^5 < y^8 < y^3 < x^6$
   Where 0 should be placed?



5. Check the equality: $2^3 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 = 1000$

6. Represent as a fraction:
   Examples: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

   $a.\ 4^{-2}$; $b.\ 3^{-3}$; $c.\ 2^{-5}$; $d.\ 5^{-2}$

7. What would be the last digit of $6^n, n = 1, 2, ...$
   What would be the last digit of $5^n, n = 1, 2, ...$
   What would be the last possible digits of $2^n, n = 1, 2, ...$

8. What should be the exponent for the equation to hold?
   $a.\ 8^* = 512$; $b.\ 2^* = 64$; $c.\ 3^* = 81$; $d.\ 7^* = 343$

9. A boy had a bag of apples. He gave 1/2 of them to his parents, 1/5 to his brother, 1/4 to his sister and the last apple he ate himself. How many apples did he originally have?

10. Come up with the problem about the distance between two objects, that can be solved by the formula, and solve it.
    Example: $d = 500 - 2.5(70 + 30)$
    Problem: Two cities are 500 miles apart. A bus and a car started moving toward each other. Speed of the car is 70 m/h, speed of the bus is 30 m/h. What would be the distance between them in 2.5 hours?
    $d = 500 - 2.5(70 + 30) = 500 - 2.5 \cdot 100 = 250\ miles$

    $a.\ d = 18 + (16 + 4) \cdot 3$ $b.\ d = 96 - 4 \cdot (56 - 40)$

    $c.\ d = 4 + 2 \cdot (12 - 7)$