Math 4 e. Class work 3.

Factorization.

In mathematics factorization is a decomposition of one number into a product of two or more numbers, or representation of an expression as a product of 2 or more expressions, which called *factors*. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$. Can you explain why?

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Or in a numerical expression:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 3 \cdot 2 \cdot 6$$

Prime numbers.

Among all natural numbers there are numbers, which are not divisible by any other number, but one and themselves.

Natural numbers greater than one that has no positive divisors other than 1 and itself are called prime numbers.

- Can a prime number be an even number?

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else.

Prime factorization of a number is the determination of the set of **prime** numbers which multiply together to give the original number. It is also known as **prime** decomposition.

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5$.

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Each prime factor is a divisor of the decomposed number, for example, 168 is divisible by two, and the result will be $168: 2 = 84 = 2 \times 2 \times 3 \times 7$:

$$2 \times \underbrace{2 \times 2 \times 3 \times 7}_{=84} = 168; \;\; 2 \cdot 84 = 168; 168 : 84 = 2$$

Product of any combination of prime factors also will be a factor (and a divisor of course). 4 is a divisor or 168, as well as 8, 6, 12, 14, 21, 24, 28, 42, 56, 84. I didn't mention 1 and 168. All factors are

1, 2, 3, 4, 6. 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168.

What are factors (divisors) of 180?

Each prime factor is a divisor, as well as product of any combinations of the prime factors:

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 45, 60, 90, 180.

Do these two numbers have common factors (common divisors)?

E can see that they are 2, 3, 4, 6, 12. The greatest common factor, GCF (and greatest common divisor GCD) is 12. How we can find GCF without writing down all possible divisors?

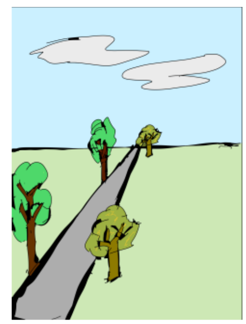
If two numbers are not prime, but don't have any common factors, they are called mutually prime numbers. For example, 8 and 9. They are not prime, but don't have any common factors.

Problem: For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly?

To solve this problem, we have to find a number which can serve as a divisor for 168 as well as for 180. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as

many kids as possible equally. To do this they have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both (168 and 180) amounts of candy simultaneously. Let's take a look at a set of all prime factors of 168 and 180. For 168 this set contains 2, 2, 2, 3, and 7. Any of these numbers as well as any of their products can be divider for 168. The same goes for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two sets have common factors. It means that both numbers are divisible by any of these common factors and any their products. The largest product is the product of all common elements. This largest product is GCD.

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168;$$
$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180;$$



Problem 2.

Along the straight road, the landscaping company is planting maples on one side and oaks on the other. They planted an oak and a maple opposite each other at the start of the road, and then planted an oak every 168 feet and a maple every 180 feet. At what distance will oak and maple be planted opposite each other again?

Multiples of 168 are

$168 \cdot 2 = 336, 168 \cdot 3 = 504,$
$672, 840, 1008, 1176, 1344, 1512, 1680, 1848, 2016, 2184, 2352, 2520, \ldots$

Multiples of 180 are

$360, 540, 720, 900, 1080, 1260, 1440, 1620, 1800, 1980, 2160, 2340, 2520 \ldots$

Now we have to find a number, which is divisible by both numbers, 168 and 180. The product of them $168 \cdot 180 = 30240$ is un answer. But is there a smaller number? Maybe the first place where oak and maple will be planted opposite each other is closer than that?

$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 168;$$
$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$
$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2520$$

$2520 : \left(\underbrace{2 \cdot 2 \cdot 2 \cdot 3 \cdot 7}_{168} \right) = 15; \quad 2520 : \left(\underbrace{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}_{180} \right) = 14$

First time where oak and maple will be planted together is 2520 feet from the beginning, and then in $2520 \cdot 2, 2520 \cdot 3$ and so on, But the least (smallest) common multiple, or LCM, is 2520.

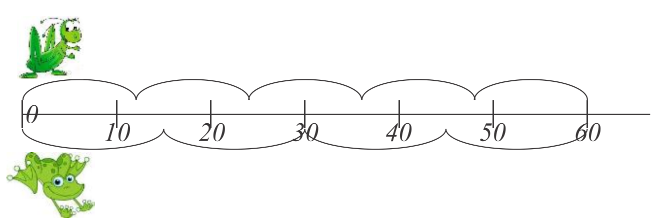
Exercise:

1. Find all prime factors of the following numbers:
   66, 28, 128, 555, 1233
   Example: $66 = 2 \cdot 3 \cdot 11$

2. Do the prime factorization of the numbers:
   34, 40, 100, 225, 1000

3. Find GCD (GCF) of
   420 and 450,
   810, 945 and 1125

4. Find LCM of
   a. 8 and 12
   b. 15, 18, and 21



5. A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet?

6. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest number of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

7. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different

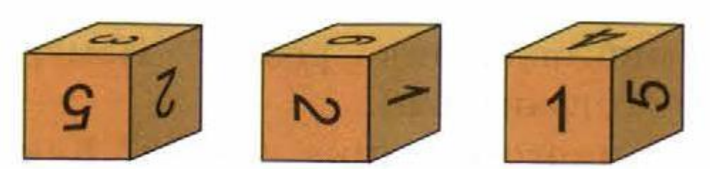
kind of flowers in each flower bed. What is the largest possible size of her square flower bed?

8. “Sweet Mathematics” sweets are sold in 12 pieces per box, and “Geometry with Nuts” sweets are sold in 15 pieces per box. What is the smallest number of boxes of both chocolates you need to buy to have equal number of both kind of chocolates?

9. Is number a divisible by number b? if yes, find the the quotient.
   $a)$ $a = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 7, \quad b = 2 \cdot 2 \cdot 11;$
   $b)$ $a = 2 \cdot 3 \cdot 5 \cdot 13, \quad b = 5 \cdot 13$
   $c)$ $a = 3 \cdot 5 \cdot 5 \cdot 11 \cdot 17, \quad b = 3 \cdot 5 \cdot 17;$
   $d)$ $a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 19 \cdot 23, \quad b = 2 \cdot 2 \cdot 3 \cdot 5$

10. Among four consecutive natural numbers will be a number
    a. Divisible by 2?
    b. Divisible by 3?
    c. Divisible by 4?
    d. Divisible by 5?

11. On each side of the cube, digits from 1 to 6 are drawn. Three positions of the cube are shown on the picture. What is the digit on the bottom of the cube in each case?



12. 3 identical books and 5 identical notebooks costs 95 dollars, but 1 same book and 2 same notebooks cost 33 dollars. How expensive are one book and one notebook?

13. Evaluate by the most convenient way:
    Example:

$$13 \cdot 11 + 13 \cdot 26 + 37 \cdot 87 = 13 \cdot (11 + 26) + 37 \cdot 87 = 13 \cdot 37 + 37 \cdot 87$$
$$= 37 \cdot (13 + 87) = 37 \cdot 100 = 3700$$

$17 \cdot 34 + 26 \cdot 17 + 13 \cdot 60;$

$4 \cdot 45 + 4 \cdot 45 + 6 \cdot 55 + 6 \cdot 45;$

14. Find LCM for numbers:

    a. 12, 15, 18

    b. 8, 12, 16.

15. Find the smallest natural number that when divided by 2, 3, 5 and 7 gives the remainder 1.