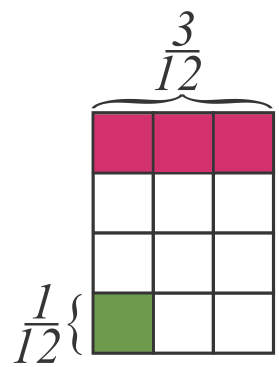
Math 4e. Class work 4.

Algebra. Fractions.

A fraction (from Latin: fractus, "broken") represents a part of a whole.

Look at the picture on the right:

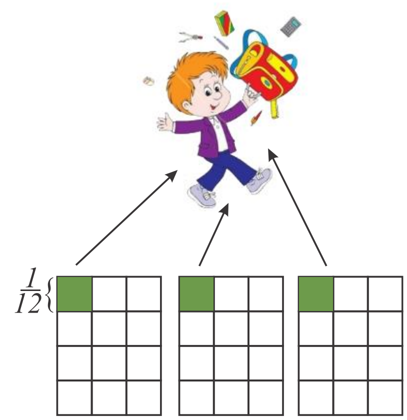


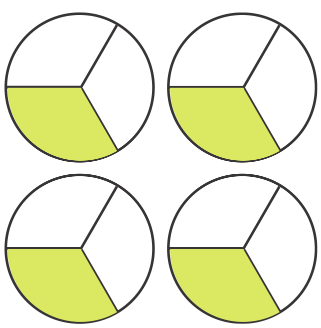
the whole chocolate bar is divided into 12 equal pieces:

$$1\,(whole\ chocolate\ bar\,)\div 12(equal\ parts)$$
$$= \frac{1\,(whole\ chocolate\ bar\,)}{12(equal\ parts)}$$
$$= \frac{1}{12}(of\ whole\ chocolate\ bar)$$
$$\frac{1}{12}+\frac{1}{12}+\frac{1}{12} = 3\times\frac{1}{12} = \frac{3}{12} = \frac{1}{4} = 3\div 12$$

To divide 3 chocolate bars between 12 kids we can give each kid $\frac{1}{12}$ of each chocolate bar, altogether

$$3\div 12 = 3\times\frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

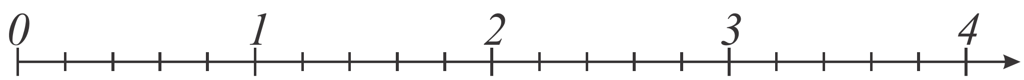




To divide 4 pizzas equally between 3 friends we will give each friend $\frac{1}{3}$ of each pizza. Each friend will get $4\div 3 = 4\times\frac{1}{3} = \frac{4}{3}$ which is exactly 1 whole pizza $(3\times\frac{1}{3} = \frac{3}{3} = 1)$ and $\frac{1}{3}$.

Mark following fractions on the number line:

$$\frac{1}{5}, \quad \frac{3}{5}, \quad \frac{3}{3}, \quad \frac{7}{5}, \quad \frac{10}{5}$$

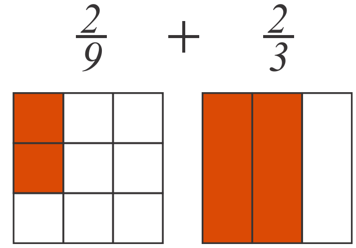


When we are talking about fraction we usually mean the part of a unit. Proper fractions are parts of a unit; improper fractions are sums of a natural number and a proper fraction. Sometimes we want to find a part of something which is not 1, but can be considered as a single object. For example, among my 30 pencils $\frac{2}{5}$ are yellow. How many yellow pencils do I have? What does it mean to find $\frac{2}{5}$ out of 30? The whole pile of all of all these pencils is a single object and we want to calculate how many pencils does a little pile of $\frac{2}{5}$ of 30 contain? $\frac{2}{5}$ is 2 times $\frac{1}{5}$, and $\frac{1}{5}$ of 30 is $30 \div 5$. So $\frac{2}{5}$ of 30 pencils will be twice more: $\frac{2}{5} \times 30 = 30 \div 5 \times 2$

Addition and subtraction of fractions with unlike denominators.

Let's try to add $\frac{2}{9}$ and $\frac{2}{3}$. What should we do? Why do we need to bring both fractions to the same denominator? We can add together only similar objects: apples to apples and oranges to oranges. Are two fractions $\frac{2}{9}$ and $\frac{2}{3}$ similar objects?

$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$, $\qquad \frac{2}{9} = \frac{1}{9} + \frac{1}{9}$



How we can add together

$$\frac{2}{9} + \frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{3} + \frac{1}{3}$$

To be able to add two fractions we have to be sure that they have the same denominator. Each $\frac{1}{3}$ is exactly the same as $\frac{3}{9}$ and $\frac{2}{3} = \frac{6}{9}$

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3}$$

Common denominator of both fractions should be the multiple of these denominators. If both numbers are prime (or mutually prime), the least common multiple is their product. If this is not the case, least common multiple is the simplest common denominator, but not the only one, any other multiple can do this task. Nominator and denominator of each fraction should be multiply by a number to bring both fractions to a common denominator.

For example,

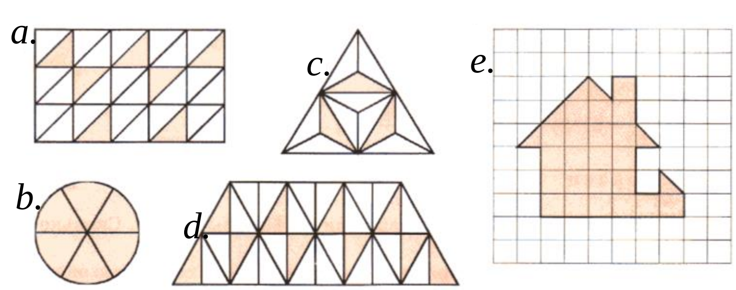
$$\frac{3}{8} + \frac{5}{12}$$

Common denominator can be $8 \cdot 12 = 96$, but 24 is smaller.

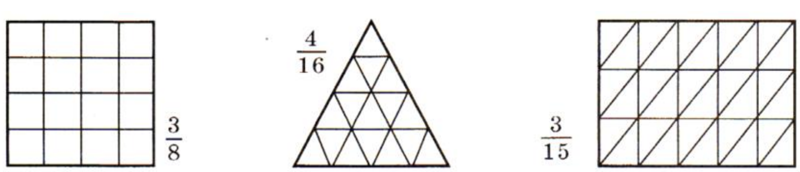
$$\frac{3 \cdot 3}{8 \cdot 3} + \frac{5 \cdot 2}{12 \cdot 2} = \frac{9}{24} + \frac{10}{24} = \frac{19}{24}$$

Exercises.

1. Write a fraction which show the shaded part of the shape:



2. Shade the corresponding part of the figure:

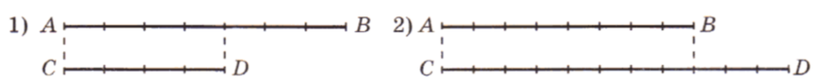


3. Compare:

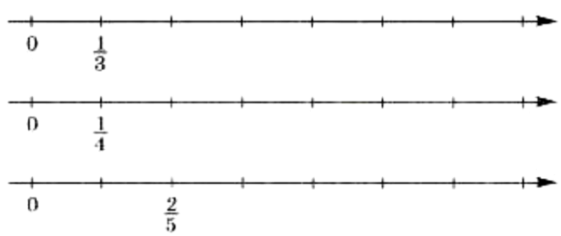
$\frac{3}{5} \quad \frac{2}{5}$ $\qquad$ $\frac{3}{5} \quad \frac{3}{8}$ $\qquad$ $\frac{3}{6} \quad \frac{1}{2}$

$\frac{1}{5} \quad \frac{5}{1}$ $\qquad$ $\frac{4}{12} \quad \frac{3}{4}$ $\qquad$ $\frac{2}{11} \quad \frac{1}{12}$

4. What part of the segment [AB] is the segment [CD]?



5. On the number lines below, mark the number 1.



6. Calculate:

$\frac{1}{5}+\frac{1}{2}=$ $\qquad$ $\frac{2}{5}+\frac{3}{10}=$ $\qquad$ $\frac{5}{9}-\frac{1}{3}=$

7.
   a. What is bigger, the number *c* or $\frac{2}{3}$ of the number *c*? Why?
   b. What is bigger, the number *b* or $\frac{3}{2}$ of the number *b*? Why?
   c. What is bigger, $\frac{2}{3}$ of a number *m* or $\frac{3}{2}$ of a number *m*? Why?

8. a. $\frac{1}{7}$ of all students in the class is 4. How many students are there in the class?

   b. $\frac{2}{5}$ of all students in a class is 10. How many students are there in a class?

9. $\frac{5}{8}$ of a number is 15. What is the number?

10. The kilogram of cookies costs 15 dollars. How much Mary paid for $\frac{4}{5}$ of the kilogram of the cookies.

11. In the school cafeteria there are 12 tables. There are 10 seats at each table. At the lunch time $\frac{4}{5}$ of all sits were occupied by students. How many students were in the cafeteria?

12. An apple worm was eating an apple. On the first day it ate half of the apple, on the second day it ate half of the rest, and on the third day it ate half of the rest again. On the fours day it ate all the leftovers. What part of the apple did it eat on the fourth day?

13. Peter spent 2 hours doing his homework. $\frac{1}{3}$ of this time, he spent doing his math homework and $\frac{1}{4}$ of the remaining time he spent on the history assignment. How many minutes did Peter spent on his history assignment and how many minutes did he spent doing his math homework?

14. Write the expression for the following problems:
    a. 3 packages of cookies cost *a* dollars. How many dollars do 5 of the same packages cost?
    b. 5 bottles of juice cost *b* dollars. How many bottles can one buy with *c* dollars?

15. Come up with the word problem which can be solved using the following expression:

$$25 - 2 \cdot 3;$$

(This expression is equivalent of 2 shorter expressions:

1. 2×3
2. $25 - 6$

16. Half of the students of the class partisipated in a spellingbee competition. One third of them bacame winners. How many students are in the class, if there are 5 winners of the spellingbee in the class?

17. Find

$$a.\ \frac{3}{4}\ of\ 12, \qquad b.\ \frac{2}{7}\ of\ 14, \qquad c.\ \frac{5}{8}\ of\ 56$$

18. There are 48 pencils of each color: blue, yellow and green pencils, 72 red pencils and 120 coloring pictures. How many identical coloring sets can be created out of these pencils and pictures?

19 There are 4 children in the family. They are 5, 8, 13, and 15 years old and their names are Julia, Peter, Mary and Ellen. What is the age of each of them if one of the girls goes to kindergarten, Julia is older than Peter, and sum of ages of Julia and Mary is divisible by 3?

20 Read the following statements:

a. All prime numbers are even numbers.
b. All odd numbers are prime numbers.
c. All prime numbers greater than 2 are odd numbers.
d. All odd numbers greater than 2 are composite numbers.

Which statement is true statement and which is false?

21. Rebecca wants to decorate the box with a birthday present for her friend Alice with a ribbon as shown in the picture. How long should the ribbon be if 90 cm should be left for the ends and the bow?