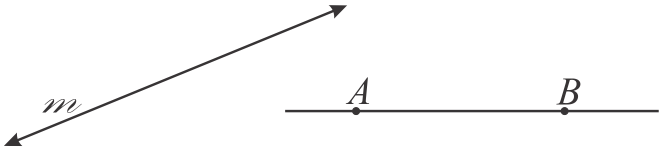
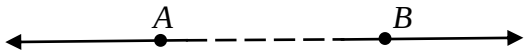
Math 4 e. Classwork 23.

Simplest geometrical shapes: segment, ray, polygon, circle.

Draw a line and mark two points on the line. All points of the straight line between two given points form a segment ([AB]).



Why the following statement can't be a definition of a segment?
Part of the straight line between two given point is called a segment.



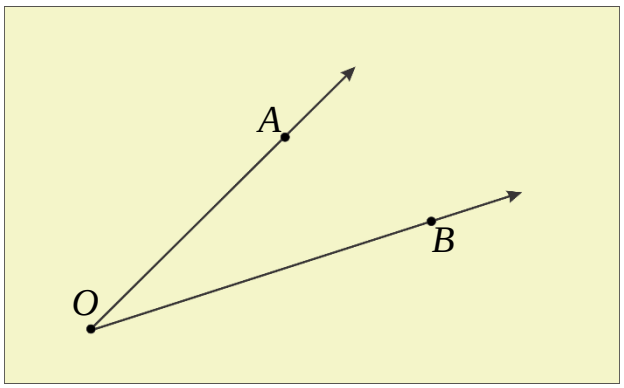
Draw a line. Mark one point on it, point A. This point produces two rays.

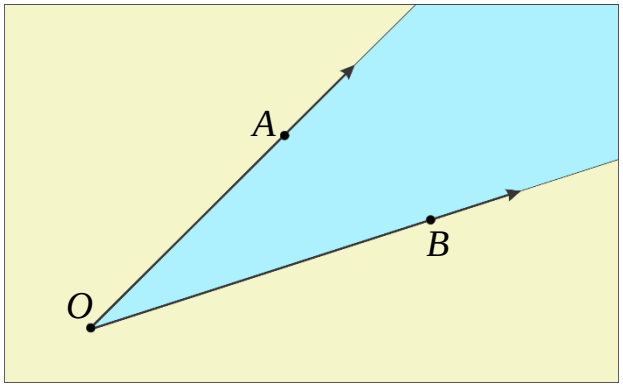
All points of a straight line on one side of a chosen point is a ray ($\overrightarrow{AB}$).

If we draw two rays from the same endpoint, we will get an angle.

- *Into how many parts does an angle divide a plane?*

We can consider an angle to be two rays or two rays and the part of the plane they limit. The difference only important when we look for the intersection of an angle and another geometrical figure.





(Intersection of two geometrical figures is a common part of these figures).

Problem 1 Draw two rays so that their intersection is

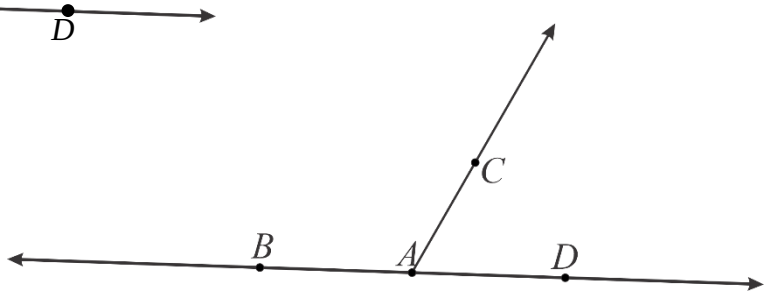
a. A point
b. A segment
c. A ray.

Problem 2. Let consider an angle as part of the plane along with two rays. Draw an angle and a line so that their intersection will be

a. A segment
b. A point
c. A ray
d. No intersection.

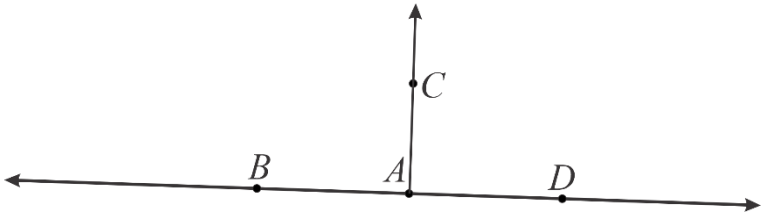
Angles' notations are usually three capital letters with the vertex letter in the middle or small Greek letter: $\angle$ABC, α.
If a point marked on a line, it produces two rays with the common vertex, therefore it's an angle. This angle has its own name: a straight angle, $\angle BAD$.

If another ray is coming from the vertex of a straight angle, it creates two more angles, $\angle CAB, \angle CAD$.



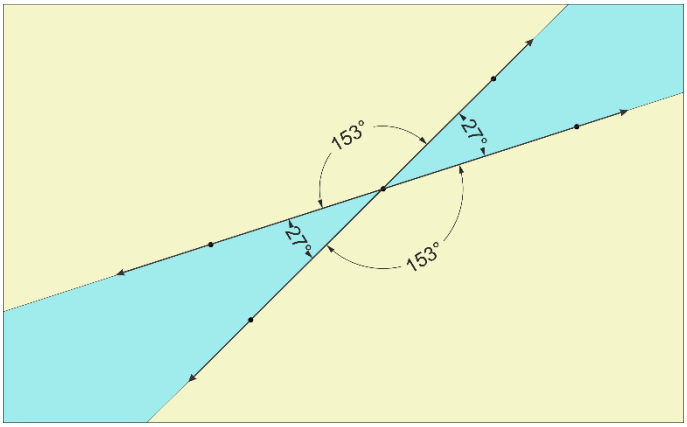
- *What can you say about these angles?*

Such angles we call supplementary angles.
There is only one angle which supplement itself to a straight angle. In this case supplementary angles are equal, and we call this angle a right angle. Measure of a straight angle is 180°, measure of a right angle is 90° in the system with 360° degrees of the full rotation.



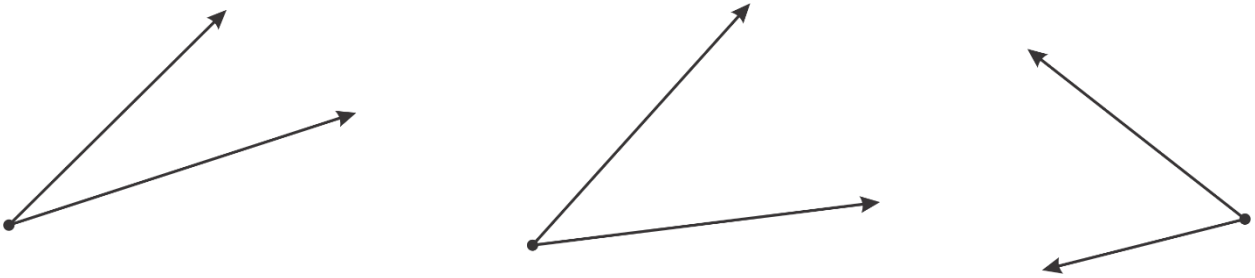
Problem 2. What will be the measure of the straight angle and right angle in the system with 400° full rotation? 200° full rotation?

Problem 3. Vertical angles (angles formed by two intersecting lines) are equal. Why?



Measuring angles.

1. Measure angles:



2. Using the protractor draw angles 20°, 30°, 60° so, that they all have one common side.

3. Using only the ruler, draw angles 30°, 45°, 80°, 90°, 120°. Measure them with protractor. How close you were you?

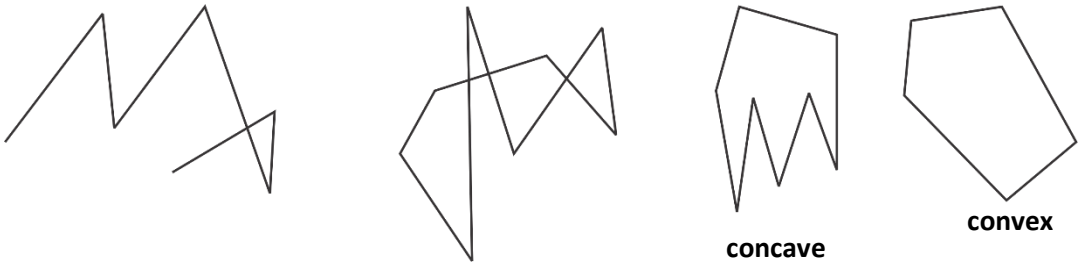
4. Draw a quadrilateral which has three right angles. What would be the fourth angle?

5. Draw a square. Draw the diagonals in this square. Which angle the diagonal forms? Why do you think so?

6. If you look at a 10-degree angle through a magnifying glass, what would be the measure of the angle you see?

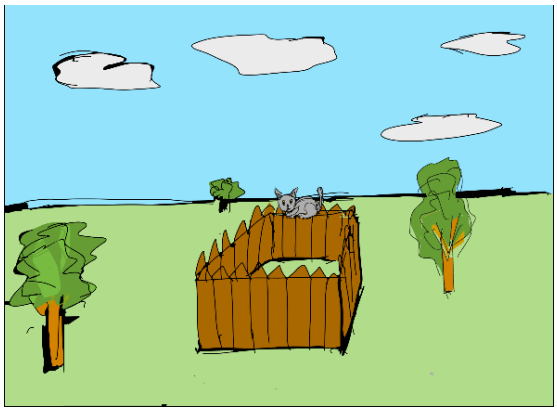
Polygons.

Draw a chain of segments, so that the last point of one segment is a first point of the next, and three consecutive points don't lie on the same line.
Draw such chain so that the last point of the last segment is the first point of the first one. We got a closed broken line. Is this a sufficient condition to get a polygon?



- *What is the difference between convex and concave polygons?*

- Polygon is a closed figure (area), two points inside of the polygon can be connected without crossing the boundaries, but one point inside and one outside can't be connected without crossing the edge of the figure.



The simplest polygon is a triangle.
Draw a triangle. Measure its angles. Add them together. What number did you get?

7. Draw the isosceles (triangle which has two equal sides)
   a. right triangle
   b. acute triangle
   c. obtuse triangle

8. Can you draw inside an isosceles triangle another isosceles triangle with the same (equal) sides? With bigger sides? In

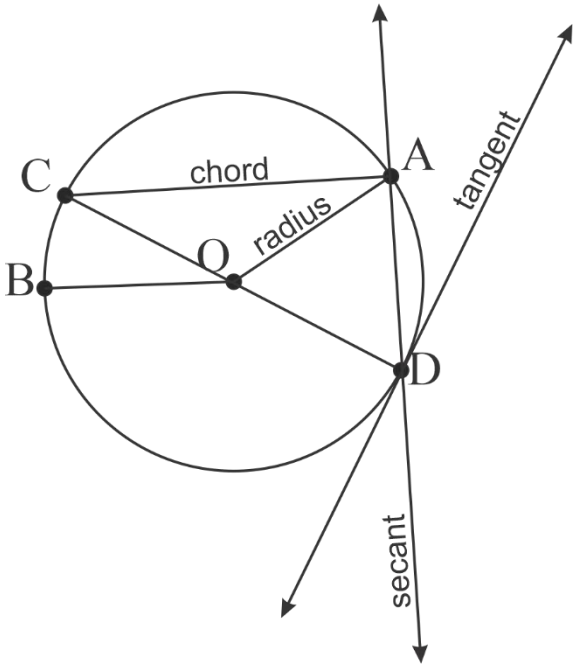
9. Draw a triangle with sides 3 cm, 5 cm and the angle between them 50°.

10. Draw a triangle with angles 30° and 50° and the side between them 7 cm. Do we need another information?

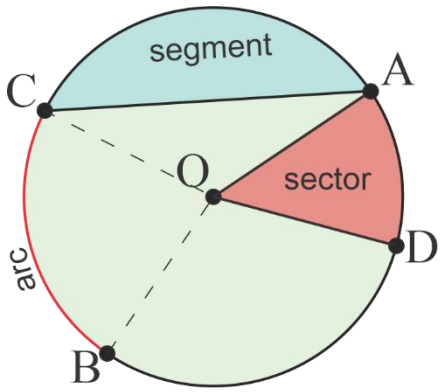
11. Into how many parts three rays can divide a plane?
12. Into how many parts two angles can divide a plane?

Circle is the set of all points in a plane that are at a same distance from a given point, the center.

Let’s define a few new words, we will used them when we will be talking about circle.

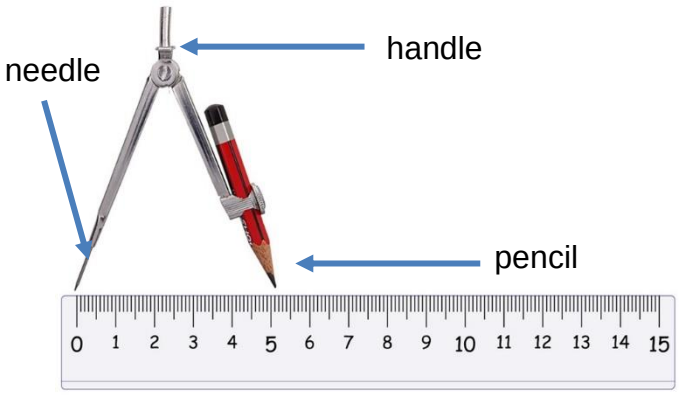


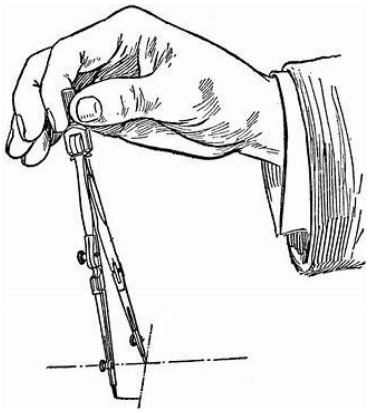
All points of a circle (without part of the plane inside it) are on the same distance from center. On the picture above, points A, B, C, and D are marked on the circle, they are on the same distance from point O, the center, as well as all other points on circle. This distance is called *radius* (plural radii). Through any two points of a circle, we can draw a line, this line will cut the circle into two parts, such line is called secant (line $\overleftrightarrow{AD}$ on the picture). The line got its name from Latin word “secare”, which means “to cut”. The segment of this line between points of its intersection with the circle is called chord. On the picture a few chords ([CA], [CD] and [AD]) are drawn. Also, in a circle there are special kind of chords, diameters. Diameter is a chord passing through the center of the circle. On the picture, chord [CD] goes through the center O. The length of any dimeter is twice the radius. Straight line can be drawn in a way that it has only one (not two) common point with a circle. Such line is called tangent line, it touches (intersects) the circle in only one point. (Of course, we can draw a circle and a straight line so, that they will not have common points, but this situation is simple).



The part of the circle (with the part of the plane inside it) limited by two radii is called a sector. Very good example of a sector is a slice of pizza. The part of a circle which is cut off from the circle by a secant and less than half of the circle, is called a circular segment. (Secant divides a circle into two parts, one is less than half and another is greater than half. Sometime, when a secant goes through the the center, it divides a cercle into two equal parts, half-circles). Part of the circular line (circle without the part of the plane inside) between two points is called an arc. Arc $\overset{\frown}{\mathrm{BC}}$, arcs $\overset{\frown}{\mathrm{CA}}, \overset{\frown}{\mathrm{AD}}$ are on the picture above. Central angle between radii for sector and arc can be measured and will be the measure for sector or arc. Angle $\angle \mathrm{AOD} = 39°, \angle \mathrm{COB} = 83°$, Arc $\overset{\frown}{\mathrm{BC}} = 83\,°$

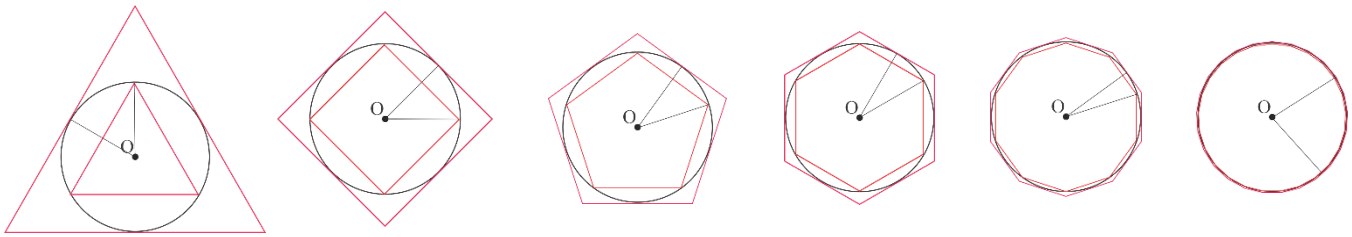
Problem 3. Draw a circle with the radius of 5 cm. Use compass. Open the compass for 5 cm. Put the needle of the compass into paper, holding the compass by its handle rotate the pencil around the needle.





The ratio of the circumference to the diameter is known as π:

$$\pi = \frac{l}{d}$$



π can be estimated by computing the ratio of the perimeters of circumscribed and inscribed polygons to the doubled radius.

For example, for the circle with the radius 5:

the perimeter of inscribed triangle can be calculated as (rounded to) 25.98 and the perimeter of circumscribed triangle as (rounded to) 51.96 (the exact technical way to do it is outside of our class, you will learn it later, in 7th and 8th grades). The diameter is 2 timce radius, so it's 10:

$$\frac{25.98}{10} < \pi < \frac{51.96}{10}; \qquad 2.598 < \pi < 5.195$$

Square:

$$\frac{28.28}{10} < \pi < \frac{40}{10}; \qquad 2.828 < \pi < 4.0$$

Pentagon

$$\frac{29.38}{10} < \pi < \frac{36.33}{10}; \qquad 2.934 < \pi < 3.635$$

Hexagon

$$\frac{30}{10} < \pi < \frac{34.64}{10}; \qquad 3 < \pi < 3.462$$

10-side polygon:

$$\frac{30.9}{10} < \pi < \frac{32.49}{10}; \qquad 3.09 < \pi < 3.249$$

25-side polygon:

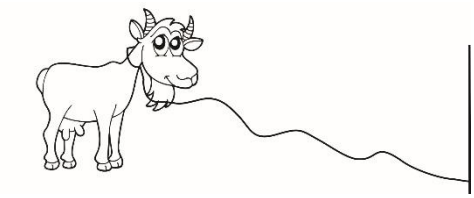
$$\frac{31.33}{10} < \pi < \frac{31.58}{10}; \qquad 3.133 < \pi < 3.158$$

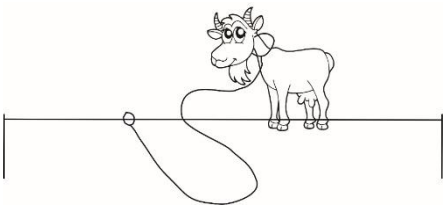
100-side polygon

$$\frac{31.41}{10} < \pi < \frac{31.42}{10}; \quad 3.141 < \pi < 3.142$$

We can continue this approximation of the ratio of circumference to the diameter of circle by the ratio of the perimeters of inscribed and circumscribed polygons to the diameter of the circle, increasing the number of polygon sides indefinitely. This ratio is called π. It's an irrational number 3.14159265358979……, but the fraction $\frac{22}{7}$ or a decimal 3.14159 (or even 3.14) is good approximation.

1. A goat is tied to a stake (or 2 poles 9 m apart) with a rope of length (3 m). What shape it will graze? Draw the picture using 1 cm for 1 m.





2. Same goat now attached to the rectangular shape 12 by 8 m.