MATH 5: HANDOUT 16
BEGINNING PROBABILITY – 1.

Beginning probability theory

We will be talking about "tests" (such as tossing a coin, rolling a die, drawing a card, etc), each of which can result in one of several possible outcomes (e.g., rolling a die can give numbers 1 through 6). If there are n possible outcomes, and they are all equally likely, then probability of getting any given one is exactly $1/n$; for example, probability of rolling a 3 on a die is $1/6$.

In most cases, we will be interested in probability of something that can be obtained in more than one way. For example, if we ask what is the probability of rolling an even number on a die, then there are 3 ways to get it: by rolling 2, 4, or 6. Each of these outcomes has probability $1/6$, so the probability of getting one of them is $1/6 + 1/6 + 1/6 = 3/6 = 1/2$. In general, if we ask what is the probability of getting one of a certain collection A of outcomes, then the answer is given by

$$P(A) = \frac{\text{number of outcomes giving } A}{\text{total number of possible outcomes}}$$

Addition rule

Suppose we are drawing a card from the deck of 52 cards and ask what is the probability of getting either queen or king. Since there are 4 queens and 4 kings, which makes it 8 cards total, we can write

$$P(\text{queen or king}) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

We can also write it as follows:

$$P(\text{queen or king}) = \frac{4+4}{52} = \frac{4}{52} + \frac{4}{52} = P(\text{queen}) + P(\text{king})$$

In general, we have the following rule:

$$P(A \text{ or } B) = P(A) + P(B)$$

if A and B can't happen together (see below).

Note that this rule only applies if A and B do not happen together. For example, there are 26 red cards in the deck, so probability of drawing a red card is $\frac{26}{52} = \frac{1}{2}$. However, if we need to get a red card or a queen, then using addition formula would give $\frac{26}{52} + \frac{4}{52} = \frac{30}{52}$, which is incorrect: this way, we have counted red queens twice. Correct answer is $\frac{28}{52}$: 26 red cards plus two black queens (no need to counte red queens –they have already been counted).

Complement rule

$$P(\text{not } A) = 1 - P(A)$$

For example, probability of drawing a queen from a deck of cards is $\frac{1}{13}$; thus, the probability of drawing something other than a queen is $1 - \frac{1}{13} = \frac{12}{13}$

Homework

1. Write each of the following expressions in the form 2^n5^k:

(a) $\dfrac{2^2 5^8}{2^5 5^3}$

(b) $(2^3)^2 10^2 5^{-3}$

(c) $\dfrac{2^8 5^{-14}}{10^{-3}}$

2. Solve the following equations:

(a) $5-2(3-x)=-2$

(b) $1-\frac{2}{3}(x+1)=x$

(c) $\dfrac{x-2}{x-4}=-2$

3. In the game of roulette, there are 37 slots, numbered 0 through 36. Of numbers 1–36, half are red, the other half are black (zero has no color). What is the probability of hitting

(a) A number between 1–12

(b) An even number other than zero

(c) A red number or zero

4. You roll two dice, one red, one black. What is the probability of rolling two ones? Of rolling a 4 and a 6?

5. The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace.

If you randomly draw one card, what is the probability of getting

(a) The queen of spades

(b) A face card (i.e., jack, queen, or king)

(c) A black king

(d) Anything but the queen of hearts

6. I had drawn a card from the deck, and it turned out to be an ace. Now I am drawing one more card from the same deck. What is the probability that it will be an ace again?

7. What is the probability that a randomly chosen person was born

(a) in January?

(b) on Feb 5?

(c) on Sunday?

*(d) On Sunday in January?[Hint: among all the people born in January, what fraction was born on Sunday?]

When doing this problem, you can ignore leap years and assume that birthdays are randomly distributed among all days of the year, so each day is equally likely; in real life it is not quite true.

8. Suppose we have a box of 500 candy of different colors and sizes. We know that there are 100 large ones and 400 small ones; we also know that there are 70 red ones, 11 of which are large. From this information, can you compute the probability that a randomly chosen candy will be either red or large (or both)?

9. If we roll two dice, what is the probability that the product of two numbers is a multiple of 3?

10. When tossing a coin, it can land head up or tails up (we will write H for head and T for tails).

(a) If we toss a coin 5 times, what is the probability that all 5 will be heads?

(b) If we toss a coin 5 times, what is the probability of getting this sequence of results: HHTHT? Is it more likely or less likely than getting all 5 heads?