

MATH 5: HANDOUT 17
BEGINNING PROBABILITY – 2.

PRODUCT RULE

Question: we roll two dice. What is the probability of rolling a 5 and a 6?

Answer: There are two ways of getting a 5 and a 6: as pair (5, 6) (5 on die number 1, 6 on die number 2) or as (6, 5) (6 on die number 1, 5 on die number 2). Thus, the answer is $\frac{2}{36}$.

Question: we roll two dice. What is the probability of getting sum of two numbers equal to 4?

Answer: there 3 ways of getting sum 4: (1, 3), (2, 2), (3, 1). Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. $(\frac{1}{2})^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or 1/10 of 1%).

Question. If toss a coin 10 times, what is the probability that all will be tails?

Answer. The same.

Question. If we toss a coin 10 times, what is the probability that **at least one** will be heads?

Answer. Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations **do not** give at least one heads: there is exactly one such combination, all tails; probability of getting this combination is, as we computed, $1/2^{10} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of getting at least one heads is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$.

PERCENTAGES AND FRACTIONS

So far we have mostly expressed probabilities as fractions. They can also be written as decimal numbers (between 0 and 1): for example, $\frac{1}{5} = \frac{2}{10} = 0.2$. It is also common to express probabilities as percentages: by definition,

$$1\% = \frac{1}{100} = 0.01$$

so $x\% = \frac{x}{100}$. For example, $3\% = \frac{3}{100} = 0.03$, and $1.5\% = \frac{1.5}{100} = \frac{15}{1000} = 0.015$.

This conversion is necessary when you multiply probabilities as the following example shows:

Question. The probability of winning in a certain game is $p = 5\%$. What is the probability of winning two times in a row?

Answer. According to multiplication rule it is $p \times p = p^2$. However, the answer $5\% \times 5\% = 25\%$ is wrong. Correct answer is $\frac{5}{100} \times \frac{5}{100} = \frac{25}{10,000} = 0.0025$.

To convert from decimals to percent, multiply by 100:

$$p = (p \times 100)\%$$

For example, $\frac{1}{5} = 0.2 = (0.2 \times 100)\% = 20\%$

HOMWORK

1. If we roll two dice, what is the probability that the product of two numbers is a multiple of 2?
2. Recall that a roulette has 37 slots: 0 through 36. Among slots 1–36, half are red, the other half black (zero has no color). What is the probability of
 - (a) getting a red (on a single run of roulette)
 - (b) getting a red, then black, then 0 (on 3 successive runs)
 - (c) getting red 15 times in a row?
 - (d) getting this sequence of colors: RRRBRBRBBRBBRBR (also of length 15)?
3. A hunter is shooting ducks. Probability of hitting a duck with one shot is $p = 1/3$.
 - (a) What is the probability of missing the duck (with one shot)?
 - (b) He makes 5 shots. What is the probability that he misses all five?
 - (c) What is the probability that out of 5 shots, he will hit at least once? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
 - (d) What is the probability that out of 5 shots, he will hit exactly once? Will this probability double if he makes 10 shots?
 - * (e) What is the probability that out of 5 shots, he will hit at least twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
 - * (f) What is the probability that out of 5 shots, he will hit exactly twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
4. Supposing that there are equal chances of a boy or a girl being born, what is the probability that at least one of the first five babies born next Saturday morning at the St. Charles Hospital will be a girl? That all five will be girls?
5. At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win. Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
 - (a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
 - (b) Same questions for five balls.
 - (c) They charge you 2 dollars for 3 balls, or 3 dollars for 5 balls. Which is a better deal? [Considering only from the point of view of the chances of winning, not the fun you are getting]