MATH 5: HANDOUT 18
CHOOSINGS AND PERMUTATIONS.

Choosing with repetitions

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet?

Answer: there are 26 possibilities for the first letter, 26 for the second, and so on — so according to the product rule, there are $(26)^3$ possible combinations.

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are allowed: same item can be used more than once (e.g., same letter may appear several times in a combination).

Then there are n^k ways to do it.

Choosing without repetitions

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet if no letter can be used more than once?

Answer: there are 26 possibilities for the first letter; after we have chosen the first letter, it leaves only 25 possibilities for the second letter; after choosing the second, we only have 24 possibilities left for the third. So the answer is $26 \times 25 \times 24$

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are not allowed: no item can be used more than once.

Then there are $n(n-1)\dots(n-k+1)$ ways of doing it (the product has k factors). This number is usually denoted

$$_kP_n = n(n-1)\dots(n-k+1)$$

Factorials and permutations

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to put n objects in some order:

$$n! = {_nP_n} = n(n-1)\cdot\dots\cdot 2\cdot 1$$

(reads n factorial).

For example: there are $52!$ ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2\cdot 3\cdot 4 = 24$, $5! = 120$, $6! = 720$

Using factorials, we can give a simpler formula for $_kP_n$:

$$_kP_n = \frac{n!}{(n-k)!}$$

For example:

$$_4P_6 = 6\cdot 5\cdot 4\cdot 3 = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = \frac{6!}{2!}$$

HOMEWORK

1. Little Bear fell down from the honey tree. His friend Porcupine, an aspiring movie director, filmed his fall. Now Porcupine plans to add sound effects to his movie. Whenever Little Bear hits a tree branch, Porcupine plans to add an exclamation: either "Oooh!" or ""Aaah!" or "Uuuh!"

(a) How many different soundtracks can Porcupine create if during his fall Little Bear collides with 2 branches?

(b) How about 3 branches? 4 branches? 5 branches?

2. Extraterrestrials from the planet Mumba-Umba use 4 letters: "A," "B," "M," and "U." Any combination of these letters makes a word in the Mumba-Umba language. (For example, AUUA and UUU are both words of the Mumba-Umba language.)

(a) How many 2-letter words are in this language? How many 3-letter words?

(b) How many 3-letter words with all letters different?

3. In a certain club of 30 people, they are selecting a president, vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?

4. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

5. How many ways are there to seat 15 students in a classroom which has 15 chairs?

6. A small theater has 50 seats. One day, all 50 seats were taken – but the people took seats at random, paying no attention to what was written on their ticket.

(a) What is the probability that everyone was sitting in the right seat (i.e., the one written in his ticket)?

*(b) What is the probability that no person was sitting in the right seat?

7. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so the pattern matches (this kind of puzzles is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long will it take you to try them all?

8. 10 people must form a circle for some dance. In how many ways can they do this?