MATH 5: HANDOUT 22
GEOMETRY 3.

Congruence tests for triangles

Recall that by definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

Congruence test 1 (**SSS** Side-Side-Side rule)**.** *If* $AB = A'B'$*,* $BC = B'C'$ *and* $AC = A'C'$ *then* $\triangle ABC \cong \triangle A'B'C'$*.*

Congruence test 2 (**ASA** Angle-Side-Angle rule)**.** *If* $\angle A = \angle A'$*,* $\angle B = \angle B'$ *and* $AB = A'B'$*, then* $\triangle ABC \cong \triangle A'B'C'$*.*

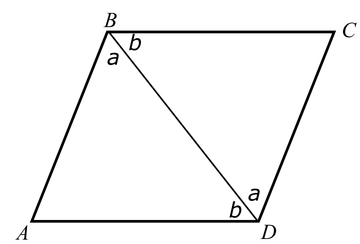
This rule is commonly referred to as ASA rule.

Congruence test 3 (**SAS** Side-Angle-Side rule)**.** *If* $AB = A'B'$*,* $AC = A'C'$ *and* $\angle A = \angle A'$*, then* $\triangle ABC \cong \triangle A'B'C'$*.*

These rules — and congruent triangles in general — are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

Theorem. *Let* $ABCD$ *be a parallelogram. Then* $AB = CD$*,* $BC = AD$*, i.e. the opposite sides are equal.*

Proof. Let us draw diagonal BD. Then the two angles labeled by letter a in the figure are equal as alternate interior angles (because $AB \parallel DC$); also, two angles labeled by letter b are also equal. Thus, triangles $\triangle ABD$ and $\triangle CDB$ have a common side BD and the two angles adjacent to it are the same. Thus, by ASA, these two triangles are congruent, so $AD = BC$, $AB = CD$. □



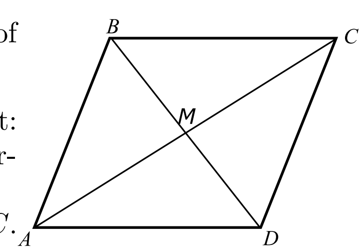
Homework

1. Solve the equation $3x + 3 = \frac{1}{2}x + 13$

2. (a) Prove that a diagonal of a rectangle cuts it into two congruent triangles.
(b) Explain why in a rectangle, opposite sides are equal.

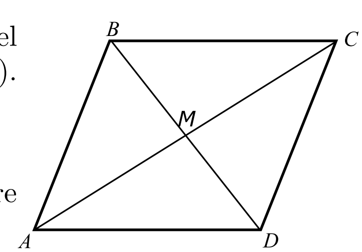
3. Let $ABCD$ be a parallelogram, and let M be the intersection point of the diagonals.

(a) Prove that triangles $\triangle AMB$ and $\triangle CMD$ are congruent. [Hint: use the parallelogram property proved in class, that in the parallelogram opposite sides are equal, and ASA.]
(b) Prove that $AM = CM$, i.e., M is the midpoint of diagonal AC.



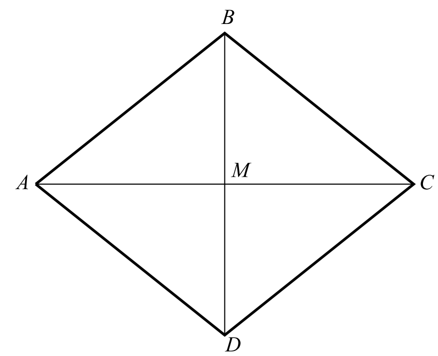
4. Let $ABCD$ be a quadrilateral such that sides AB and CD are parallel and equal (but we do not know whether sides BC and AD are parallel).

(a) Prove that triangles $\triangle AMB$ and $\triangle CMD$ are congruent.
(b) Prove that sides BC and AD are indeed parallel and therefore $ABCD$ is a parallelogram.

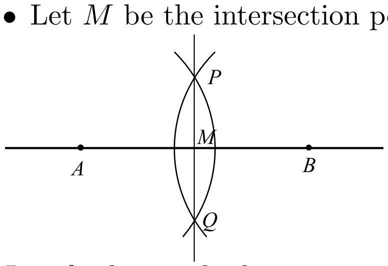


5. We know that in a rhombus $ABCD$ all sides are equal: $AB = BC = CD = AD$. Let M be the intersection point of AC and BD.
   (a) Prove that $\triangle ABC \cong \triangle ADC$
   (b) Prove that $\triangle AMB \cong \triangle AMD$
   (c) Prove that the diagonals AC and BD are perpendicular
   (d) Prove that the point M is the midpoint of each of the diagonals AC and BD.

   [Hint: after doing each part, mark on the figure all the information you have found — which angles are equal, which line segments are equal, etc: you may need this information for the following parts.]



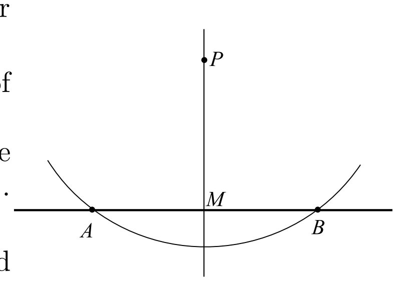
6. The following method explains how one can find the midpoint of a segment AB using a ruler and compass:
   - Choose radius r (it should be large enough) and draw circles of radius r with centers at A and B.
   - Denote the intersection points of these circles by P and Q. Draw the line PQ.
   - Let M be the intersection point of lines PQ and AB. Then M is the midpoint of AB.



   Justify this method, i.e., prove that so constructed point will indeed be the midpoint of AB? You can use the defining property of the circle: for a circle of radius r, the distance from any point on this circle to the center is exactly r.[Hint: $APBQ$ is a rhombus, so we can use the knowledge about the rhombus from the previous problem.]

7. The following method explains how one can construct a perpendicular from a point P to line l using a ruler and compass:
   - Choose radius r (it should be large enough) and draw circle of radius r with center at P.
   - Let A, B be the intersection points of this circle with l. Find the midpoint M of AB (using the method of the previous problem). Then MP is perpendicular to l.

   Justify this method, i.e., explain why so constructed MP will indeed be perpendicular to l?



8. Let $ABCD$ be a parallelogram, and let BE, CF be perpendiculars from B, C to the line AD.
   (a) Prove that triangles $\triangle ABE$ and $\triangle DCF$ are congruent.
   (b) Show that the area of parallelogram is equal to height$\times$base, i.e. $BE \times AD$.