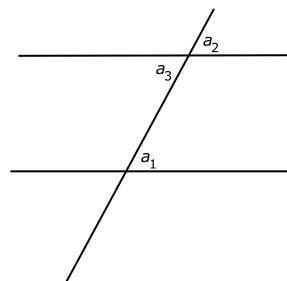


## MATH 5: HANDOUT 25 GEOMETRY REVIEW

### Parallel lines and alternate angles

If one has two parallel lines and intersects both of them by a third line as shown in the figure to the right, then angles labeled by letters  $a_1, a_3$  (alternate interior angles) will be equal. Conversely, if these two angles are equal, then the lines must be parallel.



### Congruence tests for triangles

Two triangles are congruent if the corresponding sides are equal and corresponding angles are equal:  $\triangle ABC \cong \triangle A'B'C'$  is the same as  $AB = A'B', BC = B'C', AC = A'C', \angle A = \angle A', \angle B = \angle B', \angle C = \angle C'$ .

**Congruence test 1** (SSS Rule). *If three sides of one triangle are equal to corresponding sides of another triangle, then the triangles are congruent.*

**Congruence test 2** (ASA Rule). *If two angles and a side between them of one triangle are the same as two angles and the side between them in another triangle, then the triangles are congruent.*

**Congruence test 3** (SAS Rule). *If two sides and an angle between them of one triangle are the same as two sides and an angle between them in another triangle, then the triangles are congruent.*

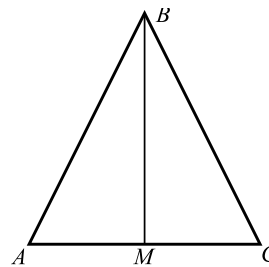
### Sum of angles of a polygon

Sum of angles of a triangle is  $180^\circ$ . Sum of angles of an  $n$ -gon is  $(n - 2) \times 180^\circ$ . For example, for a pentagon we get  $3 \times 180^\circ = 540^\circ$ .

### Isosceles triangle

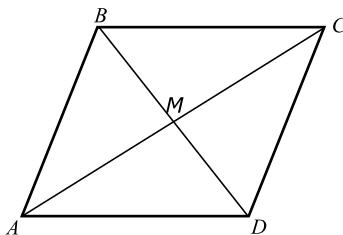
Let  $ABC$  be a triangle in which two sides are equal:  $AB = BC$  (such a triangle is called *isosceles*). Let  $M$  be the midpoint of the side  $AC$ , i.e.  $AM = MC$ . Then

1. Triangles  $\triangle ABM$  and  $\triangle CBM$  are congruent.
2. Angles  $\angle A$  and  $\angle C$  are equal
3.  $\angle AMB = 90^\circ$



### Parallelograms

**Definition.** A parallelogram is a quadrilateral in which opposite sides are parallel.



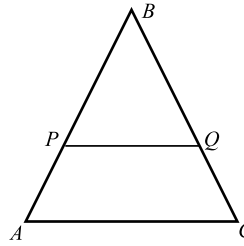
#### Properties:

1. In a parallelogram, opposite sides are equal. Conversely, if in a quadrilateral opposite sides are equal, then it is a parallelogram.
2. In a parallelogram, diagonally opposite angles are equal  $\angle A = \angle C, \angle B = \angle D$
3. The intersection point  $M$  of the two diagonals is the midpoint for each of them.

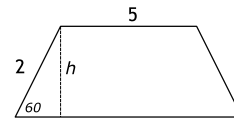
## Homework

1. Let  $ABCD$  be a rectangle.
  - (a) Explain why the triangles  $ABC$  and  $ABD$  are congruent.
  - (b) Explain why in a rectangle, the two diagonals are equal.

2. Triangle  $ABC$  is isosceles, and  $\angle A = 50^\circ$ .
  - (a) Find the other angles in the triangle.
  - (b) In the same triangle, line  $PQ$  is parallel to  $AC$ . Find angles  $\angle P$ ,  $\angle Q$ .
  - (c) Prove that  $BP = PQ$ .

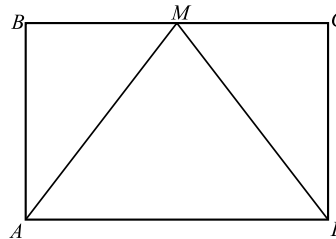


3. The figure to the right shows some of the angles and sides in a trapezoid. The height  $h$  of this trapezoid is equal to  $\sqrt{3}$ . Find all other angles, sides and area of the trapezoid. (Hint: you will need Pythagorean theorem!)

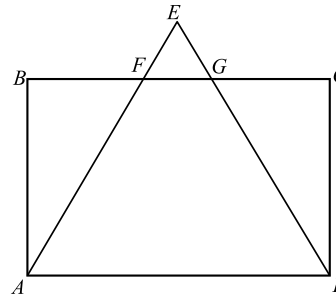


4. Explain how one can construct a rectangle with given sides  $a, b$ , using only ruler and compass (but not the protractor!).

5. In the figure to the right,  $ABCD$  is a rectangle, and  $M$  is the midpoint of  $BC$ . Prove that then triangle  $AMD$  is isosceles.



6. In the figure to the right,  $ABCD$  is a rectangle, and  $AE = DE$ . Prove that then  $BF = CG$ .



7. The following list shows some numbers, written by words in the language of some Pacific island nation. Each next number is equal to the previous one plus 2. Can you determine what these numbers are?
  - thabung ke nua lo
  - thabung ke nua vak
  - libenyita ke nua khasa
  - libenyita ke nua kun
  - libenyita ke nua thabung
  - libenyita ke nua thabung ke nua lo
  - libenyita ke nua thabung ke nua vak