MATH 5: HANDOUT 3
ALGEBRAIC EXPRESSIONS

In mathematics and other sciences we often use letters instead of numbers. Usually it is done to show that certain relationship will work for all numbers. Letters are also commonly used for unknown values. These letters are called *variables*.

Expressions involving both numbers and variables are called *algebraic expressions*.

Examples: $3a$; $7b + 8$; $357 + 10x$; $(65z - 459) \div 4$

In algebraic expressions we omit the sign of multiplication between a number and a variable. Instead of $7 \times b$ we write $7b$, instead of $10 \times z$ we write $10z$. In products, a number goes first, and then goes a variable. We do not write $k \times 10$, we write $10k$.

Using variables, we can write the basic rules for addition an multiplication as follows:

$a + b = b + a$ commutative law for addition

$a + (b + c) = (a + b) + c$ associative law for addition

$ab = ba$ commutative law for multiplication

$a(bc) = (ab)c$ associative law for multiplication

$a(b + c) = ab + ac$ distributive law

These laws can be used for simplifying calculations and rewriting expressions in a simpler form. For example:

$$2x + 3 + 5 \times (x + 1) = 2x + 3 + 5x + 5 \qquad \text{"opening the parentheses"}$$
$$= 2x + 5x + 3 + 5 = (2 + 5)x + 8 = 7x + 8$$

The operation we did in the last line — combining terms $2x$ and $5x$ into a single term $7x$ — is very commonly used; it is called “collecting the like terms”. Note, however, that it is only possible if the terms contain the same variable: we can not collect like terms in an expression like $2x + 7y$.

We also discussed the method for solving simple equations. The main idea is that we start with a given equation and then transform it, making it simpler and simpler, until at the end we can find the value of the variable. In particular:

- Given an equation, we can or subtract add to both sides the same number. For example, we can replace equation $3x + 5 = 20$ by $3x = 15$ (obtained by subtracting 5 from both sides of the original equation)
- We can multiply or divide both sides of an equation by the same number. For example, we can replace $3x = 15$ by $x = 5$ (obtained by dividing both sides by 3).

Homework problems on back

Homework

Please try to do as many of the problems below as you can, and bring completed solutions with you to next class (do not forget to put your name on it!). Some of these problems are similar to those we have discussed in class; some are new. It is OK if you can not solve some problem — but do not give up before making an effort, maybe putting the problem away and coming back to it later — which means you have to start the homework early.

Please always write solutions on a separate sheet of paper. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Compute:

(a) $1\frac{7}{8} \times \frac{18}{5}$ (b) $2\frac{4}{7} \div \frac{4}{21}$ (c) $\frac{13}{7} - \frac{7}{13}$

2. Find the values of these algebraic expressions:
(a) $78 + 3x$ for $x = 8$; 2.3; and $\frac{2}{3}$;
(b) $54 \div (x - 7)$ for $x = 8.5$; 13; and 11;

3. Using the laws above, try to rewrite each of the expressions below in the simplest possible form, by collecting the like terms if possible.

(a) $2x + 7 + 5x + 2 + 3x$ (b) $3x + 9 + 5xy + 2xy + 3$
(c) $2x + 16 + 10xy + 5x + 3$ (d) $2a + 1 + 3(a + 2)$

4. Solve the following equations.

(a) $x + 12 = 34$ (b) $24 - x = 10$ (c) $2x = 96$
(d) $3x + 2 = 44$ (e) $5(x + 4) = 45$

5. Cut a triangle into 4 triangles, any two of which have a common boundary (not just a point, but a whole segment!).

6. Below are some examples from a multiplication table in an unknown language. All of the products are numbers less or equal than 20.
pe × nei = nei la nei
nei × hato = liomu la pe
hato × hato = nei la tano
pe × pe = nei
pe × tano = liomu
hato × * = liomu la tano
* × pe = liomu la nei
What numbers should be there in place of *?

7. Marina has a bag of M&M candy. There are three colors in the bag: red, green, and brown. She knows that if you draw 100 pieces of candy form the bag (it is a very large bag), then among them there must be candy of all three colors. How many pieces of candy can there be in her bag? try to find the maximal number possible.