MATH 5: HANDOUT 15
DIFFERENCE OF SQUARES. SQUARE ROOT. PYTHAGOREAN THEOREM.

DIFFERENCE OF SQUARES

There is an important formula that allows you to factor a difference of squares:

$$a^2 - b^2 = (a - b)(a + b)$$

SQUARE ROOTS

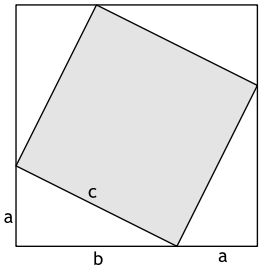
Square root of a is a number whose square is equal to a. For example: square root of 25 is 5, because $5^2 = 25$.

- Notation: square root of number a is commonly denoted $\sqrt{a}$.
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b}$ is **not** equal to $\sqrt{a} + \sqrt{b}$.

Pythagorean theorem: In a right triangle with legs a, b and hypotenuse c, one has

$$a^2 + b^2 = c^2 \qquad \text{ot} \qquad c = \sqrt{a^2 + b^2}.$$

Proof: Consider the following picture:



In this square, the total area is

$$(a + b) \times (a + b) = a \times (a + b) + b \times (a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

On the other hand, the area of each triangle is $\frac{1}{2}ab$, and the area of shaded square is c^2. Thus, we get

$$a^2 + 2ab + b^2 = 4 \times \tfrac{1}{2}ab + c^2$$

which gives $a^2 + b^2 = c^2$. □

For example, in a square with side 1, the diagonal has length $\sqrt{2}$.

It is possible — but not easy — to find a right triangle where all sides are whole numbers. The easiest such triangle is the triangle with sides $3, 4, 5$.

POWER $\frac{1}{2}$

We know how to raise numbers into whole powers:

$$a^n = a \times \cdots \times a.$$

But what is $a^{\frac{1}{2}}$?

Example: Let's try to figure out what $4^{\frac{1}{2}}$ is:

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{\frac{1}{2}+\frac{1}{2}} = 4^1 = 4.$$

We can see that $4^{\frac{1}{2}}$ must be a number, such that if we multiply it by itself, we get 4. But this is just a square root of 4! So, we get:

$$4^{\frac{1}{2}} = \sqrt{4}.$$

In general, this is also true:

$$a^{\frac{1}{2}} = \sqrt{a}.$$

HOMEWORK

1. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

2. Can you find whole numbers a, b such that $a^2 - b^2 = 17$? [Hint: use the formula we talked about in class, and think what $a - b$ and $a + b$ must be.]

3. Find the following square roots. If you can not find the number exactly, at least say between which two whole numbers the answer is, e.g., between 5 and 6.

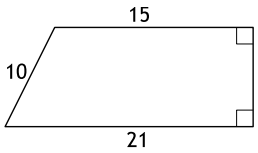
(a) $\sqrt{16}$ (b) $\sqrt{81}$ (c) $\sqrt{10,000}$ (d) $\sqrt{10^8}$ (e) $\sqrt{50}$

4. Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?

5. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?

6. Find $\sqrt{2^6 \times 7^2}$; $\sqrt{\frac{1}{16}}$; $\sqrt{\frac{4}{9}}$;

7. Find the height and area of the figure below. Lengths of three sides are given; the two marked angles are right angles.



8. The side of an equilateral triangle is 1 m. Find its height and the area.

9. Take some positive number $x < 100$ and using calculator (or computer) calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result x and repeat the same calculation with the new x. Do it 10 times. Then take the result and square it. What did you get? Try to do the same thing starting with different numbers. Is it surprising?