MATH 5: HANDOUT 18
CHOOSINGS AND PERMUTATIONS.

CHOOSING WITH REPETITIONS

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet?

Answer: there are 26 possibilities for the first letter, 26 for the second, and so on — so according to the product rule, there are $(26)^3$ possible combinations.

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are allowed: same item can be used more than once (e.g., same letter may appear several times in a combination).

Then there are n^k ways to do it.

CHOOSING WITHOUT REPETITIONS

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet if no letter can be used more than once?

Answer: there are 26 possibilities for the first letter; after we have chosen the first letter, it leaves only 25 possibilities for the second letter; after choosing the second, we only have 24 possibilities left for the third. So the answer is $26 \times 25 \times 24$

The same method of counting can be applied in more general situation: suppose we need to choose k items from a collection of n so that

- Order matters: choosing A, then B is different from choosing B, then A.
- Repetitions are not allowed: no item can be used more than once.

Then there are $n(n-1)\dots(n-k+1)$ ways of doing it (the product has k factors). This number is usually denoted

$$_kP_n = n(n-1)\dots(n-k+1)$$

FACTORIALS AND PERMUTATIONS

Recall from last time: if we are choosing k objects from a collection of n so that a)order matters and b)no repetitions allowed, then there are

$$_kP_n = n(n-1)\dots \quad (k \text{ factors})$$

ways to do it.

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to put n objects in some order:

$$n! = {}_nP_n = n(n-1)\cdot \dots \cdot 2 \cdot 1$$

(reads n factorial).

For example: there are $52!$ ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2 \cdot 3 \cdot 4 = 24$, $5! = 120$, $6! = 620$

Using factorials, we can give a simpler formula for $_kP_n$:

$$_kP_n = \frac{n!}{(n-k)!}$$

For example:

$$_4P_6 = 6 \cdot 5 \cdot 4 \cdot 3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6!}{2!}$$

Homework

1. In a certain club of 30 people, they are selecting a president, vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?

2. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

3. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

4. A small theater has 50 seats. One day, all 50 seats were taken – but the people took seats at random, paying no attention to what was written on their ticket.

(a) What is the probability that everyone was sitting in the right seat (i.e., the one written in his ticket)?

*(b) What is the probability that no person was sitting in the right seat?

5. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so the pattern matches (this kind of puzzles is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long will it take you to try them all?

6. (a) How many 5s are there in the prime factorization of the number $100!$? How many 2s?

(b) In how many zeroes does the number $100!$ end?

7. 10 people must form a circle for some dance. In how many ways can they do this?