MATH 5: HANDOUT 20
MONTY-HALL PROBLEM.

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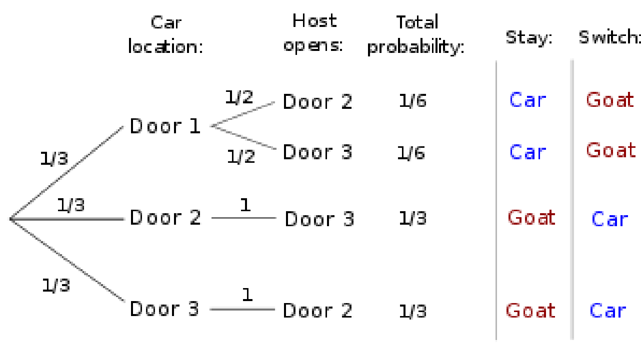
Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

A car and two goats are arranged behind three doors, with the position of the car being random and uniformly distributed among the three doors, and then the player initially picks a door. Assuming the player's initial pick is Door 1 (the same analysis applies for any other door the player picks), then according to the problem statement in which the host must always open a door after the player chooses one, then there are three equally likely cases:

- The car is behind Door 3 and the host must open Door 2: Probability $= 1/3$
- The player originally picked the door hiding the car. The game host must open one of the two remaining doors randomly, so there are two subcases
  - The car is behind Door 1 and the host opens Door 2: Probability $= 1/6$
  - The car is behind Door 1 and the host opens Door 3: Probability $= 1/6$
- The car is behind Door 2 and the host must open Door 3: Probability $= 1/3$

These cases and the total probability of each of them occurring are shown in the figure below. If the host has opened Door 3 switching wins in the $1/3$ case where the car is behind Door 2 and loses in one $1/6$ subcase where the car is behind Door 1, hence **switching wins with probability 2/3**.



Homework

1. In a group of 100 students, 28 speak Spanish, 30 speak German, 42 speak French; 8 students speak Spanish and German, 10 speak Spanish and French, 5 speak German and French and 3 students speak all 3 languages. How many students do not speak any one of the three languages?
[Note: when it says that 28 students speak Spanish, this includes the 8 who speak Spanish andGerman; similarly for all other combinations.]

2. Suppose I have a standard die (6 faces on a cube, numbered 1 through 6).
  (a) What is the probability that when we roll the die once, the number will be less than 5?
  (b) What is the probability that when we roll the die once, the number will be less than 7?
  (c) What is the probability that when we roll the die twice, at least one result will be a 6?
  (d) What is the probability that when we roll the die twice, at least one result will be a 7?
  (e) What is the probability that when we roll the die three times, all the results will be odd?

3. (a) What is the probability that if we roll 2 dice, the sum will be at most 7?
  (b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him $1. Otherwise A loses and he pays to B $1. Would you prefer to play for A or for B in this game?
  (c) How to adjust the payments to make this game fair?

4. (a) What is the probability that if we roll 3 dice, all the numbers will be different?
  (b) A and B are playing the following game. They roll 3 dice; if all numbers are different, A wins, and B pays him $2. Otherwise A loses and he pays to B $3. Would you prefer to play for A or for B in this game?
  (c) How to adjust the payments to make this game fair?

5. (a) Two numbers are randomly chosen among 1, 2, and 3, one after the other (repeats are allowed). What is the chance that both numbers are the same?
  (b) Two numbers are randomly chosen among 1, 2, and 3, one after the other. What is the chance that they will be in strictly increasing order? (Strictly increasing means the second number must be greater than the first, they are not allowed to be equal.)

***6.** Here is another question similar to the Monty Hall question discussed today. You know that the family next door has two children. You met one of them, and he is a boy. What is the probability that the other one is a boy, too?