# MATH 5: HANDOUT 21
GEOMETRY 1.

Today we are starting a new topic: geometry. We mostly reviewed previous material, namely:
1. Lines, angles; measurement of angles. Acute and obtuse angles.
2. Triangles. Sum of angles of a triangle is equal to 180. Opposite the larger angle is the larger sde; if two angles are equal, then opposite sides are also equal.
3. Rectangles and squares. Opposite sides in a rectangle are equal.
4. Areas. Area of a rectangle with sides l, w is $l \times w$. Area of a right triangle with legs a, b is $\frac{1}{2}ab$.

and added some new material: parallel lines and parallelograms.

Definition: Parallel lines are lines which go "in the same direction", without intersecting.
An important property of parallel lines is one has two parallel lines and intersects both of them by a third line as shown in the figure to the right, then angles labeled by letters a_1, a_2 will be equal. Conversely, if these two angles are equal, then the lines must be parallel.
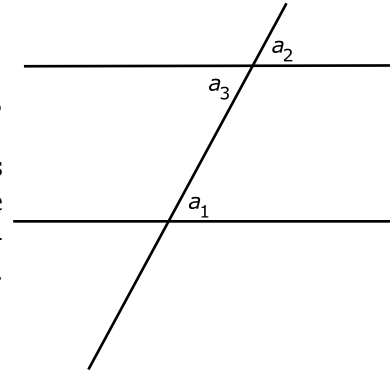
Remind also that when two lines intersect, the two angles across from each other are equal; for example, in the figure above, $\angle a_2 = \angle a_3$ (such a pair of angles is called *vertical angles*). Thus, in for parallel lines intersected by a third line, angles a_1 and a_3 are also equal (such angles are called *alternate interior angles*). Again, this rule also works in the opposite direction: if we have two lines intersected by a third one, and alternate interior angles are equal, then the two lines are parallel. This is commonly used to prove that two lines are parallel.
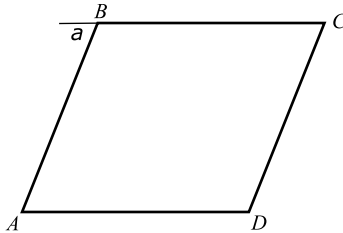
Definition: A parallelogram is a quadrilateral in which opposite sides are parallel.
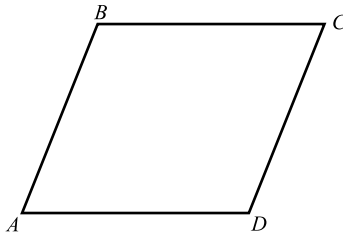

Parallelograms have a number of interesting properties, which we will study later.

1. Is it true that any rectangle is also a parallelogram? Is it true that any parallelogram is a rectangle? Try to argue as carefully as you can.

2. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$ [Hint: see figure below]


3. Show that the previous problem also works in the other direction: if in a quadrilateral, diagonally opposite angles are equal: $\angle A = \angle C$, $\angle B = \angle D$, then the quadrilateral must be a parallelogram.

4. Cut two identical paper triangles (the easiest way to do it is to fold a sheet of paper in two and then cut). Can you put these two triangles together so that they form a parallelogram? Will your method always work? Why?

5. In the figure below, $ABCD$ is a parallelogram, and $\angle\angle DAB = 40°$. Can you find all other angles in the figure?
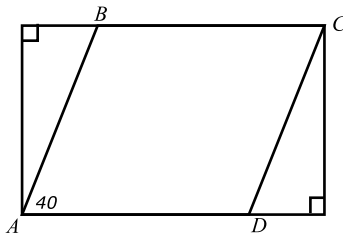
6. The reflection law states that the angles formed by the incoming light ray and the reflected one with the surface of the mirror are equal: $\angle a = \angle b$
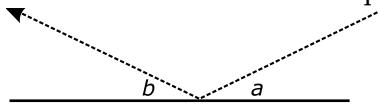
Using this law, show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one:
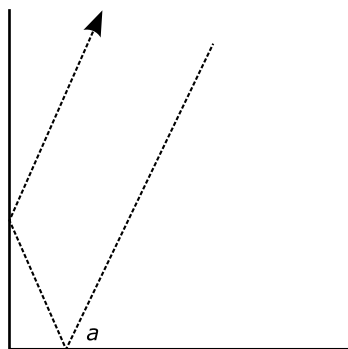
[Hint: find the angle which each of these lines form with the horizontal]

This property – or rather, similar property of corners in three dimensions — is widely used: reflecting road signs, tail lights of a car, reflecting strips on clothing are all constructed out of many small reflecting corners so that they reflect the light of a car headlamp exactly back to the car.

7. Antonia and Gabi took a 9-mile trip in a rowboat. There was only one pair of oars, so they took turns rowing (however, they didn't time how long each of them was rowing, so it could happen that one had rowed longer than the other). Antonia could row at the speed of 3 miles per hour; Gabi could only do 2 miles per hour. It took them 3.5 hours to complete the trip. Can you find out how long each of them was rowing?