MATH 6
ASSIGNMENT 5: TRUTH TABLES AND LOGIC LAWS

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TRUTH TABLES

Logical variables: take value True (T) or False (F).
Basic logic operations:

NOT ($\text{NOT } A$): true if A is false, and false if A is true.

A	$\text{NOT } A$
T	F
F	T

AND ($A \text{ AND } B$): true if both A, B are true, and false otherwise

OR ($A \text{ OR } B$) ***inclusive or***: true if at least one of A, B is true, and false otherwise.

XOR ($A \text{ XOR } B$) ***exclusive or***: true if different, false if the same.

IF (as in "if A, then B; written $A \implies B$): if A is false, automatically true; if A is true, it is true only when B is true

Logic operations can be combined, e.g. $(A \text{ OR } B) \text{ AND } C$.

Truth tables: *

A	B	$A \text{ XOR } B$
T	T	F
T	F	T
F	T	T
F	F	F

A	B	$A \text{ AND } B$
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$A \text{ OR } B$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Truth tables are useful in solving the problems about knights and knaves. Here is a typical problem: on the island of knights and knaves you meet two inhabitants, Zed and Alice. Zed tells you, 'I am a knight or Alice is a knave.' Alice tells you, 'Of Zed and I, exactly one is a knight.' We could solve it by making the following table:

Zed	Alice	Z is a knight or A is a knave	Of Z and A, exactly one is a knight
knight	knight	T	F
knight	knave	T	T
knave	knight	F	T
knave	knave	T	F

LOGIC LAWS

We can combine logic operations, creating more complicated expressions such as $A \text{ AND}(B \text{ OR } C)$. As in arithmetic, these operations satisfy some laws: for example $A \text{ OR } B$ ***is the same as*** $B \text{ OR } A$. Here are two other laws:

$$\text{NOT}(A \text{ AND } B) \text{ is the same as } (\text{NOT } A) \text{ OR}(\text{NOT } B)$$

$$A \implies B \text{ is the same as } (\text{NOT } B) \implies (\text{NOT } A)$$

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.

1. On the island next to he island of knights and knaves there are 3 kinds of people:
knights, who always tell the truth
knaves, who always lie
normal people, who sometimes lie and sometimes tell the truth
On that island, you meet 3 people, A, B, and C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:
A: I am normal
B: That is true
C: I am not normal
What are A, B, and C?

2. Check whether $A \implies B$ and $B \implies A$ are equivalent, by writing the truth table for each of them.

3. Check that $A \implies B$ is equivalent to $(\text{NOT } A) \text{ OR } B$ (thus, "if you do not clean up your room, you will be punished" and "clean up your room, or you will be punished" are the same).

4. A teacher tell the student "If you do not take the final exam, you get an F". Does it mean that
(a) If the student does take the final exam, he will not get an F
(b) If the student does not get an F, it means he must have taken the final exam.

5. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
(a) $(A \text{ OR } B) \text{ AND} (A \text{ OR } C)$
(b) $A \text{ OR} (B \text{ AND } C)$.

6. Define a new logical operation, XOR (exclusive or) as follows: $A \text{ XOR } B$ is true if exactly one of A, B is true, and false otherwise.
(a) Write the truth table for $A \text{ XOR } B$.
(b) Can you express XOR using only AND, OR, and NOT (that is, write a formula equivalent to $A \text{ XOR } B$ using only AND, OR, and NOT)? ***Hint: create truth table with columns:*** A, B, $\bar{A}$, $\bar{B}$, $A\bar{B}$, $\bar{A}B$, **and think .... which logical operations can give you XOR output.**

7. (a) Write truth tables for formulas $A \text{ AND} (B \text{ OR } C)$ and $(A \text{ AND } B) \text{ OR } C$ (hint: there will be 8 rows in the table). Are these formulas equivalent (i.e., do they always give the same answer)?
(b) The waiter in a restaurant tells you: "our fixed price dinner includes soup and appetizer or salad." Denoting
$A =$your dinner will include soup
$B =$your dinner will include appetizer
$C =$your dinner will include salad
what would be the correct way to write his statement using letters A, B, C and logical operations AND, OR?

8. You probably know Lewis Carroll as the author of *Alice in Wonderland* and other books. What you might not know is that he was also a mathematician very much interested in logic, and had invented a number of logic puzzles. Here is one of them:
You are given 3 statements.
(a) All babies are illogical.
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are despised.
Can you guess what would be the natural conclusion from these 3 statements?