MATH 6
ASSIGNMENT 8: SETS CONTINUED

Counting

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \dots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(see problem 6 below).

Product rule

If we need to choose a pair of values, and there are a ways to choose the first value and b ways to choose the second, then there are ab ways to choose the pair.

For example, a position on a chessboard is described by a pair like a4; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 64$ possible positions.

It works similar for triples, quadruples, $\dots$. For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, we get $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences we can get.

1. Let $A = [1, 3] = \{x \mid 1 \le x \le 3\}$, $B = \{x \mid x \ge 2\}$, $C = \{x \mid x \le 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

2. Long ago, in some town a phone number consisted of a letter followed by 3 digits (e.g. K651). How many possible phone numbers could there be in that town?[Note: digits could be zero, so a number like X000 was allowed.]

3. If we roll 3 dice (one red, the other white, and the third one, black), how many combinations are possible? How many combinations in which the sum of values is exactly 4?

4. A **subset** of a set A is a set formed by taking some (possibly all) elements of A; for example, the set $\{2, 4, 6, 8\}$ is a subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set which contains no elements at all and S itself).

Can you guess the general rule: if set S has n elements, how many subsets does it have?

5. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?

(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. In this problem, we denote by $|A|$ the number of elements in a finite set A.

(a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.

*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$.

7. Draw the following sets on the number line:

(a) Set of all numbers x satisfying $x \le 2$ and $x \ge -5$;

(b) Set of all numbers x satisfying $x \le 2$ or $x \ge -5$

(c) Set of all numbers x satisfying $x \le -5$ or $x \ge 2$

8. For each of the sets below, draw it on the number line and then describe its complement:
a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c)$(0, 5) \cup (2, \infty)$ where

$[a, b] = \{x \mid a \le x \le b\}$ is the interval from a to b (including endpoints),
$(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
$[a, \infty) = \{x \mid a \le x\}$ is the half-line from a to infinity (including a),
$(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)