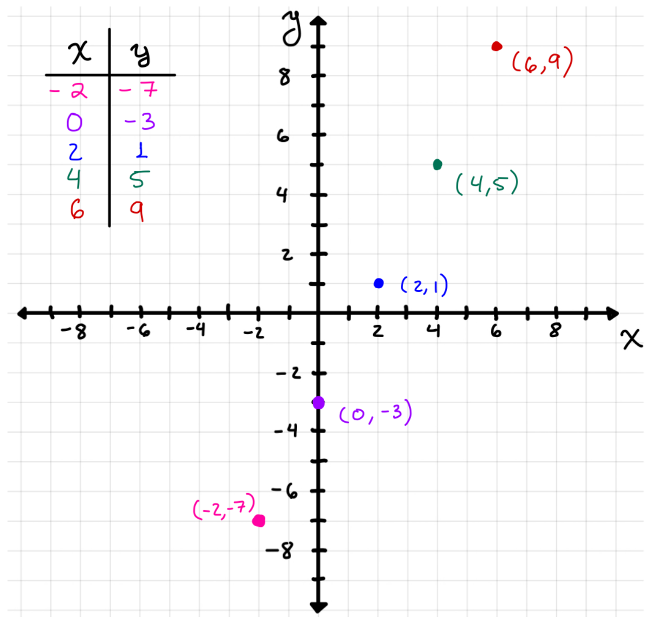
MATH 6: HANDOUT 14
COORDINATE PLANE AND COORDINATE GEOMETRY

If we want to specify a particular location in a map, we usually can do this very easily by determining its latitude and its longitude. In other words, we are specifying its **coordinates**. This is one example of what is known as a coordinate plane. In a coordinate plane, we usually have two perpendicular axes, which we label as the x-axis (horizontal) and the y-axis (vertical). Then, to specify any point, we just need to give its coordinates as (x_i, y_i).

For example, consider the following set of pairs of x and y:



Can you spot a relationship between the points marked in the coordinate plane? If you look closely, you will notice that if we join the points, we will get a perfect line. This happens because all of these pairs of numbers satisfy the following relationship

$$y = 2x - 3$$

You should check that each of the pairs satisfies the equation when you substitute theirs values in x and y.

In general, any relationship of the type

$$y = mx + b$$

will define a line in the coordinate plane, which is why equations of this type are called **linear equations**. In this equation for a line, we need to specify two numbers: m and b. The first number m is called the **slope** of the curve, and it tells us how STEEP the curve is. The second number b is called the y-intercept. As its name suggests, it tells us at which value of the y-axis our line will cross.

A line from two points: If we take two points in the coordinate plane, there will always be a straight line connecting the two of them. To find the equation of this line we need to find the slope m and the y intercept. Let's consider the two points of interest, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. We know that both points satisfy the equation of the **same** line:

$$y_1 = mx_1 + b \tag{1}$$
$$y_2 = mx_2 + b \tag{2}$$

Notice that in these equations, x_1, x_2, x_1 and x_1 are no longer variables. They represent specific numbers. If we want to find the slope m, then we can subtract one equation from the other to get

$$y_1 - y_2 = m(x_1 - x_2) \Rightarrow m = \frac{y_1 - y_2}{x_1 - x_2}$$

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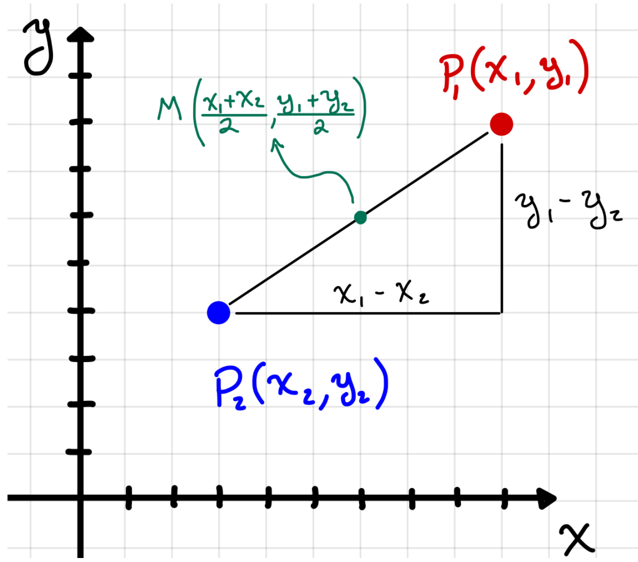

This gives us the slope of our curve, and it gives us an interpretation for it. It is telling us how much the line increases in the vertical direction per each increase in the horizontal direction. In other words, it tells us how steep is the line.

Midpoint of a segment: If we have two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the midpoint of the segment connecting these two points is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Parallel and perpendicular lines: Parallel lines are defined by having the same slope $m_1 = m_2$. In perpendicular lines, the slopes of the two lines are related by $m_1 = -1/m_2$.

Homework

1. Find the equation of a line with slope 2 and y-intercept -3.

2. What is the equation of the y-axis?

3. What is the equation of the line that passes through points $(3, 2)$ and $(2, 1)$?

4. A line l has slope $\frac{3}{5}$. What is the slope of a line parallel to l? What is the slope of a line perpendicular to l?

5. What is the equation of a line with slope $m = 2$ and containing the point $(1, 0)$?

6. Find k if $(1, 9)$ is on the graph of $y - 2x = k$.

7. Find k if $(1, k)$ is on the graph of $5x + 4y - 1 = 0$

8. Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex?

9. Show that the quadrilateral with the vertices (-1,-2), (4, -1), (5,4), (0,3) is a rhombus. Show that the diagonals are perpendicular.

10. The vertices of a triangle are $A(4, 3)$, $B(6, -1)$, $C(-2, -5)$. L,M are midpoints of BC and CA. Find the coordinates of L and M and show that $LM = \frac{1}{2}BA$

***11.** A hiker climbs a hill. He starts at 9am and reaches the summit at 4pm. The next day, he returns and starts again at 9am and reaches the base of the mountain at 4pm. Show that there exists a point on the hill where he stood at exactly the same time each day.