MATH 6: HANDOUT 15
COORDINATE GEOMETRY II

Equation of a line. Last week, we learned that we can express a line in the coordinate plane through the following linear equation

$$y = mx + b,$$

where m stands for the slope of the line and b stands for the y-intercept.

If we know two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ through which the line passes, we can find the slope m by doing

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

Midpoint of a segment: If we have two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the midpoint of the segment connecting these two points is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Parallel and perpendicular lines: Parallel lines are defined by having the same slope $m_1 = m_2$. In perpendicular lines, the slopes of the two lines are related by $m_1 = -1/m_2$.

Distance between two points: In order to find the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the coordinate plane, we can make use of the Pythagorean theorem.

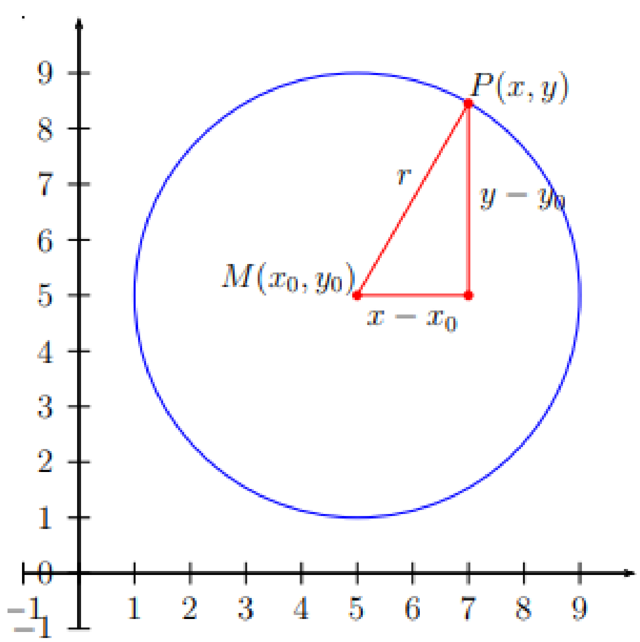
If we connect the two points by a straight line, then we can make a right triangle where this line is the hypotenuse. The legs will then be a horizontal and a vertical line. From this, we can use the Pythagorean theorem to find that the distance d between the two points is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Equation of a circle: A circle is defined as all the points in the coordinate plane that have equal distance to a fixed point $M(x_0, y_0)$. The distance to this point is what we call the RADIUS of the circle. The equation that describes a circle centered at $M(x_0, y_0)$ with a radius of r is given by

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

Can you see any similarity with the equation of the distance between two points?



HOMEWORK

1. Find the equation of the line through $(1,1)$ with slope 2.

2. Find the equation of the line through points $(1,1)$ and $(3,7)$.

3. Consider the following system of linear equations:

$$\begin{cases} 6x - 5y = -3 \\ x + \ \ y = 5 \end{cases}$$

(a) Solve the system of linear equations to find a value for x and y.
(b) Rewrite each of the equations in the form of $y = mx + b$.
(c) Graph each of these lines in a coordinate plane and find their intersection point.
(d) What can you say about the intersection point and the system of linear equations?

4. Let l_1 be the graph of $y = x + 1$, l_2 be the graph of $y = x - 1$, m_1 be the graph of $y = -x + 1$, and m_2 be the graph of $y = -x - 1$.
(a) Find the intersection point of l_1 and m_1; Label this point A and write down its coordinates.
(b) Find the intersection point of l_2 and m_2; Label this point B and write down its coordinates.
(c) Find the midpoint of AB and write down its coordinates.
(d) Let C be the intersection point of l_1 with m_2, and D be the intersection point of l_2 with m_1. What kind of quadrilateral is $ABCD$?
(e) Are l_1 and l_2 parallel? Explain why or why not?

5. (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y-1)^2 - 1 = 0$.
(c) Draw the graph of the equation $(x+2)^2 + (y+3)^2 = 4$.
(d) Draw the graph of the equation $xy = 0$.

6. (a) 3 points $A(0,0)$, $B(1,3)$, $D(5,-2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?
(b) 3 points $A(0,0)$, $B(2,3)$, $D(4,1)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?
(c) 3 points $A(0,0)$, $B(1,5)$, $D(3,-2)$ are vertices of a parallelogram ABCD. What are the coordinates of the point C?
(d) Can you guess the general rule: if $A(0,0)$, $B(b_1,b_2)$, $D(d_1,d_2)$ are 3 vertices of a parallelogram, what are coordinates of point C?

7. Consider the triangle ΔABC with the vertices $A(-2,-1)$, $B(2,0)$, $C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.