MATH 6
LOGIC II. LOGIC VARIABLES AND TRUTH TABLES

On this class we will focus on learning about logic variables, truth tables and how to use these two to simplify complicated logic problems.

Logical variables: take value True (T) or False (F).
Basic logic operations:

NOT (for example, $\text{NOT}\, A$): true if A is false, and false if A is true.

AND (for example $A \text{ AND } B$): true if both A, B are true, and false otherwise

OR (for example $A \text{ OR } B$): true if at least one of A, B is true, and false otherwise. Sometimes also called "inclusive or" to distinguish it from the "exclusive or" described below

As in usual algebra, logic operations can be combined, e.g. $(A \text{ OR } B) \text{ AND } C$.

Truth tables: If we have a logical formula involving variables $A, B, C, \dots$, we can make a table listing, for every possible combination of values of $A, B, \dots$, the value of our formula. For example, the following is the truth table for OR:

A	B	$A \text{ OR } B$
T	T	T
T	F	T
F	T	T
F	F	F

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.

They are also useful in solving the problems about knights and knaves. Recall the problem from last time, with two inhabitants, Zed and Alice. Zed tells you, 'I am a knight or Alice is a knave.' Alice tells you, 'Of Zed and I, exactly one is a knight.' We could solve it by making the following table:

Zed	Alice	Z is a knight or A is a knave	Of Z and A, exactly one is a knight
knight	knight	T	F
knight	knave	T	T
knave	knight	F	T
knave	knave	T	F

HOMEWORK

When doing the homework, remember to write arguments, not just answers!

As before, some of the questions of this assignment take place on the island of Knights and Knaves, described in the previous homework.

1. On the island of Knights and Knaves, you meet two inhabitants: Carl and Bill. Carl says, "I and Bill are both knights or both knaves." Bill claims, "Only a knave would say that Carl is a knave." [Hint: first, rewrite Bill's claim in an easier to understand form.]

(Continued on reverse)

2. On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel and Peggy. Bob says that it's not true that Peggy is a knave. Mel says that Peggy is a knight or Bob is a knave. Peggy claims, "Both I am a knight and Bob is a knave."

3. On the island of Knights and Knaves, you meet three inhabitants:Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, 'Of Joe and I, exactly one is a knight.' Joe claims, 'Bozo and I are different.'

4. Define a new logical operation, XOR (exclusive or) as follows: $A \text{ XOR } B$ is true if exactly one of A, B is true, and false otherwise.

(a) Write the truth table for $A \text{ XOR } B$.

(b) Can you express XOR using only AND, OR, and NOT (that is, write a formula equivalent to $A \text{ XOR } B$ using only AND, OR, and NOT)?

5. (a) Write truth tables for formulas $A \text{ AND}(B \text{ OR } C)$ and $(A \text{ AND } B) \text{ OR } C$ (hint: there will be 8 rows in the table). Are these formulas equivalent (i.e., do they always give the same answer)?

(b) The waiter in a restaurant tells you: "our fixed price dinner includes soup and appetizer or salad." Denoting

A =your dinner will include soup

B =your dinner will include appetizer

C =your dinner will include salad

what would be the correct way to write his statement using letters A, B, C and logical operations AND, OR?

6. Once upon a time there was a king who had many beautiful daughters. There were many dangerous tigers in his kingdom as well. A knight came to the king. He wanted to marry a princess. The king decided to test how clever the knight was. He offered the following test. There were three doors leading to three rooms. In one of the rooms, there was a princess waiting for the knight. In another, there was a a hungry tiger. The third room was empty.

The king also placed the signs on the doors — but the sign on the door of princess' room was true, the sign on the tiger's door was false, and the sign on the door of the empty room could be either false or true.

Here are the signs:

Room I	Room II	Room III
Room III is empty	The tiger is in room I	This room is empty

Which door should the knight open?

7. A certain convention numbered 100 politicians. Each politician was either crooked or honest. We are given the following two facts:

- At least one of the politicians was honest.
- Given any two of the politicians, at least one of the two was crooked.

Can it be determined from these two facts how many of the politicians were honest and how many of them were crooked?

8. Solve the following equations:

(a) $2x - 22 = 3(1 - x)$ (b) $1 - \frac{2}{7}x = \frac{1}{7}x$ (c) $1 - 8(1 - x) = 7x - 8$