Math 6d: Homework 5

HW#5 is due October 28; submit to Google classroom 15 minutes before the class time.
Please, write clearly which problem you are solving and show all steps of your solution.

Logic variables are basic statements labeled with A, B, C, ..., that can be evaluated as T or F.

For example, ***A = "Bill is a knight"*** and ***C = "Carl is a knight"*** can be evaluated (answered) as True or False.

Logic operations AND, OR, NOT combine variables (statements) in more complicated statements. A **truth table** for a logic operation evaluates all possible combinations of two variables that are combined by the logic operation.

Examples: Operation AND: A **AND** B = "Bill is a knight **AND** Carl is a knight"
Operation OR: A **OR** B = "Bill is a knight **OR** Carl is a knight",
Operations NOT: **NOT** A = " Bill is not a knight".

Truth tables:

A	B	A **AND** B
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	F

A	**NOT** A
T	F
F	T

A	B	A **OR** B
T	T	T
T	F	T
F	T	T
F	F	F

Instructions: In problems 1 and 2, you need to (a) write the obvious conclusion from the given statements; and (b) justify the conclusion by writing a chain of arguments that leads to it. It may help to write the given statements and conclusion by logical formulas (denoting which are used by letters *A*, *B*, … connected by logical operations *OR*, *AND*, …).

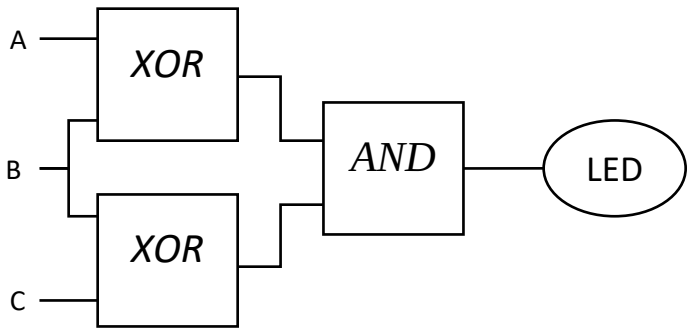
Homework questions

1. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore …

2. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore …

3. Using only *AND*, *NOT*, and *OR*, produce a three-input *AND* circuit, i.e. the output is False unless all three inputs are Ttrue. (You do not have to use all three circuit elements.)

4. The diagram right shows a circuit constructed of 3 logical chips (each chip has two inputs and one output; we draw them so that the input lines are on the left and the output line is on the right). Can you determine for which values of inputs the LED will light up? (Hint: this is the same writing a truth table for some formula…For example, If A is set to 1 and B is set to 0, what will be the output value of the XOR chip? 1 stands for T, 0 stands for F)



Note that the wires connecting each of the chips and LED to the power source are not shown.

5. Let us consider a new logical operation, called *NAND*, which is defined by the following truth table:

A	*B*	*A NAND B*
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that *A NAND B* is equivalent to *A NAND B* (this explains the name *NAND* for 'not and')
(b) Show that *A NAND A* is equivalent to *NOT A*
(c) Write the truth table for (*A NAND B*) *NAND* (*A NAND B*)
(d) Write the truth table for (*A NAND A*) *NAND* (*B NAND B*)
(e) Show that any logical formula which can be written using *AND*, *OR*, *NOT* can also be written using only *NAND*.

6. On the island of knights and knaves, you meet two inhabitants: X and Y. X says 'Y is a knave'. Y says 'X is a knave'. Who is a knight and who is a knight?