Math 6d: Homework 9

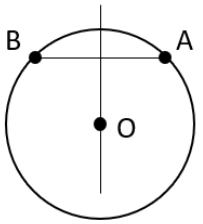
HW#9 is due December 2nd; submit to Google classroom 15 minutes before the class time.

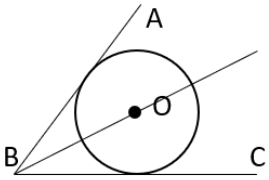
Please, write clearly which problem you are solving and show all steps of your solution.

Summary from the classwork

- **Operations** we can do using a ruler and compass.
  1. Construct the midpoint of a given segment *AB*.
  2. Construct the perpendicular bisector of segment *AB*, i.e. a line that goes through the midpoint of *AB* and is perpendicular to *AB*. (also a bisector to a chord on a circle)
  3. Given a line l and a point *A* on l, construct a perpendicular to the line l through *A*.
  4. Given a line l and a point *P* outside of l, construct a perpendicular to the line l through *P*.
  5. Given an angle *AOB*, construct the angle bisector (i.e., a ray *OM* such that $\angle AOM \cong \angle BOM$)

- **Important constructions and definitions**
  - ***Center of a circle:*** If two points *A, B* are on a circle, then the center of this circle lies on the perpendicular bisector to *AB* (i.e., a line that goes through the midpoint of *AB* and is perpendicular to *AB*).
  - ***Circle inscribed in an angle*** (circle inside in an angle). If a circle is inscribed in the angle *ABC*, then the center of this circle lies on the angle bisector.
  - ***Circle inscribed in a triangle*** (circle inside in a triangle). Construct 1) two angle bisectors that define the center of the circle and 2) a $\perp$ line to one of the sides that passes through the center of the circle.
  - ***Surcumscribed circle*** (circle around a triangle). Construct 1) two symmetry lines (bisectors) of two sides that define the center of the circle and 2) radius from center of the circle to one of the triangle's vertices.





- **Solution format** for every problem (*what we need to include):*
  A) ***Give a recipe*** (procedure) for constructing the required figure using only a ruler and compass
  B) ***Analysis.*** Prove or explain why our recipe does give the correct answer

Homework questions

All of the construction problems should be done by only using a ruler and a compass.

1. Given a circle, find its center. (Hint: construct 2 chords, then construct their bisectors)

2. Given a triangle *ABC*, construct a circle inscribed in the triangle. (Hint: construct angle bisectors)

3. Two concentric circles, where the circles have the same center and one has a larger radius, are crossed by a line at consecutive points A, B, C, and D. Prove that AB = CD.

4. Six grasshoppers sit on a road. Every minute one grasshopper jumps 1 foot in one direction (along the road), and another grasshopper jumps 1 foot in the opposite direction. If initially the grasshoppers were at positions 1ft, 2ft, …, 6ft (measured from some point on the road), is it possible that after some time they all will all gather at the same place on the road?