Math 6d: Homework 18

HW#18 is due March 3; submit to Google classroom 15 minutes before the class time.

Please, write clearly which problem you are solving and show all steps of your solution.

Sets: counting

- We use $|A|$ to denote the number of elements in a set A (if this set is finite). For example, if $A = \{a,b,c,\ldots,z\}$ is the set of all letters of the English alphabet, then $|A| = 26$.
- If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$
  For example, if there are 13 girls and 15 boys in the class, then the total is 28.
- If the sets do intersect, the rule is more complicated:
  $$|A \cup B| = |A| + |B| - |A \cap B|$$

Sets: product rule

- If we need to choose a pair of values, and there are a- ways to choose the first value and b- ways to choose the second, then there are ab ways to choose the pair.
  For example, a position on a chessboard is described by a pair like *f4*; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 8^2 = 64$ possible positions.

- It works similarly for triples, quadruples, …
  For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences.

Homework questions

1. Let set A contains the numbers 1,2,3 written in set notation as $A = [1,3] = \{x|1 \le x \le 3\}$, set B has elements colled x that are all greater or equal to 3, written in set notation as $B = \{x|x \ge 3\}$, and set C contains elements x that are all greater or equal to 1.5, written in set notation as $C = \{x|x \le 1.5\}$. Draw on a number line the following sets (one number line per set):
   (a) $\bar{A}$ (b) $\bar{B}$ (c) $\bar{C}$
   (d) $A \cap B$ (e) $A \cap C$
   (f) $A \cap (B \cup C)$ (g) $A \cap B \cap C$

2. For each of the sets below, draw it on the number line and then describe its complement (everything not in the set):
   (a) $[0 , 2]$ **(b)** $(-\infty , 1] \cup [3 , \infty)$ **(c)** $(0 , 5) \cup (2 , \infty)$, where

   $[a , b] = \{x \mid a \le x \le b\}$ is the interval from a to b (including endpoints),
   $(a , b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
   $[a , \infty) = \{x \mid a \le x\}$ is the half-line from a to infinity (including a),
   $(a , \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a).

3. Long ago, in some town, a phone number consisted of a letter followed by 3 digits (e.g. K651). How many possible phone numbers could there be? [Note that digits could be zero, i.e. X000 is allowed.]

4. If we roll 3 dice (one red, the other white, and the third black), how many possible combinations are there? How many combinations give the sum of values to be exactly 4?

5. In this problem, we use $|A|$ to denote the number of elements in a finite set A. We know that for two sets A and B, we have $|A \cup B| = |A| + |B| - |A \cap B|$

   Can you come up with a similar rule for three sets: that is write a formula for $|A \cup B \cup C|$ which uses $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$

6. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer, chess, neither, both?