HW#23 is due April 7; submit to Google classroom 15 minutes before the class time.
Please, write clearly which problem you are solving and show all steps of your solution.

Arithmetic sequences

A sequence of numbers is an *arithmetic sequence* if the difference between consecutive terms is the same number, a common difference, let's call it d. For example, consider the sequence:

$$1, 5, 9, 13, 17, \ldots$$

The first term in the sequence is: $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$. What is the nth term? For example, what is a_{100}

$a_1 = 1$
$a_2 = a_1 + d = 1 + 4 = 5$
$a_3 = a_2 + d = a_1 + 2d = 1 + 2 \times 4 = 9$
$a_4 = a_3 + d = a_1 + 3d = 1 + 3 \times 4 = 13$
$\ldots$
$\boldsymbol{a_n = a_1 + (n-1)d}$
So $a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$

Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:
$$a_n = a_{n-1} + d$$
$$a_n = a_{n+1} - d$$

Add the two expressions above and we have: $2a_n = (a_{n-1} + d) + (a_{n-1} - d)$
$$2a_n = a_{n-1} + a_{n-1}$$

Dividing by 2 both sides:
$$a_n = \frac{a_{n-1} + a_{n-1}}{2}$$

To find the common difference between 2 terms a_s and a_t: $\quad d = \frac{a_s - a_t}{s-t}$

Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing order, and in decreasing order:

$$S = a_1 + a_2 + a_3 + \cdots + a_n$$
$$S = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$$

1

Adding up:
$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

We notice that:

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$$

$$2S = (a_1 + a_n) \times n$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

Homework questions

1. Write the first 5 terms of the arithmetic sequence for which $a_3 = 7$ and $d = 12$

2. What are the first 2 terms for the arithmetic sequence $a_1, a_2, -9, -2, 5, \ldots$

3. If for an arithmetic sequence $a_{10} = 131$ and $d = 12$, what is a_1? [Hint: use the equation for $a_n = a_1 + (n-1)d$ … and solve for a_1]

4. If for an arithmetic sequence $a_5 = 27$ and $a_{27} = 60$, find the first term and the common difference. [Hint: use an equation to find d first]

5. Find the common difference in an arithmetic sequence if the 9th term is 18 and the 11th term is 44.

6. Find the sum of the first 100 terms of an arithmetic sequence if $a_1 = 10$ and $a_{100} = 150$.

7. Find the sum of all odd numbers from 1 to 2011.

8. Can you continue the following sequence: $-2, 1, 6, 13, 22, \ldots$
   [Hint: look at the differences of successive terms!] Write at least 3 more terms.

   Review from coordinate geometry:
9. Find the shortest distance from the origin (0, 0) to a line given by the equation $y = -2x + 8$. Start by sketching the graph of the line.

10. Compute the area of the rectangle $ABCD$ if A is at (0, 0), B at (2, 3), and D at (−6, 8).
    [This can be done in more than one way.]