Math 6d: Homework 25

HW#23 is due April 28; submit to Google classroom 15 minutes before the class time.

Please, write clearly which problem you are solving and show all steps of your solution.

Geometric sequence (progression)

A sequence of numbers is a geometric progression if the next number in the sequence is the current number times a constant called the common ratio, let's call it q. For example, let's consider the sequence:

6, 12, 24, 48, ...

- The first term in the sequence is $b_1 = 6$, the second is $b_2 = 6 \times 2 = 12$, and so on.
- The common ratio is $q = 2$. Indeed $b_3 = b_2 \times q = 12 \times 2 = 24$ and $b_4 = 24 \times 2 = 48$.
- What is the n^{th} term? For example what is b_{10}?

$b_1 = 6$
$b_2 = b_1 \times q = 6 \times 2 = 12$
$b_3 = b_2 \times q = (b_1 \times q) \times q = b_1 \times q^2 = 6 \times 2^2 = 24$
$b_4 = b_3 \times q = (b_1 \times q^2) \times q^2 = b_1 \times q^3 = 6 \times 2^3 = 48$
....
$b_n = b_1 \times q^{n-1}$
So $b_{10} = b_1 \times q^9 = 6 \times 2^9 = 6 \times 512 = 3072$

Sum

There is a formula for the sum of the first n- terms of a geometric progression:

$$S_n = b_1 + b_2 + b_3 + \cdots + b_n = b_1 \times \frac{(1 - q^n)}{1 - q}$$

- To prove this, we write the sum and we multiply it by q:

$$S = b_1 + b_2 + b_3 + \cdots b_{n-1} + b_n$$
$$qS = qb_1 + qb_2 + qb_3 + \cdots + qb_{n-1} + qb_n$$

Remember that $qb_{n-1} = b_n$, so that the last term is $qb_n = q \times (b_1 \times q^{n-1}) = b_1 \times q^n$.

$$qS = b_2 + b_3 + b_4 + \cdots b_n + b_1 q^n$$

We subtract S from each side:

$$qS - S = b_2 + b_3 + \cdots + b_n + b_1 q^n - (b_1 + b_2 + b_3 + \cdots b_{n-1} + b_n)$$

All terms cancel, except $b_1 q^n$ and b_1 so that:

$$qS - S = b_1 q^n - b_1$$
$$(1 - q)S = b_1 q^n - b_1$$
$$S = \frac{b_1 q^n - b_1}{1 - q} = \frac{b_1(1 - q^n)}{1 - q}$$

Homework questions

1. Write the first 5 terms of a geometric progression if $b_1 = -20$ and $q = \frac{1}{2}$.

2. What are the first 2 terms of the geometric progression: b_1, b_2, 24, 36, 54, ...?

3. What is the common ratio of the geometric progression: $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \ldots$? What is b_{10}? What is b_{100}?

4. Simplify:
$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}}$$

5. What is the sum: $1 - 2 + 2^2 - 2^3 + 2^4 - 2^5 + \cdots - 2^{15}$?

6. What is the sum: $1 + x + x^2 + x^3 + x^4 + x^5 + \cdots + x^{100}$?

7. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50^{th} term?

8. If we put one grain of wheat on the first square of the chessboard, two on the second, then four, eight,. . , approximately how many grains of wheat will there be? (You can use $2^{10} = 1024 \approx 10^3$). Can you estimate the total volume of all this wheat? Compare with the annual wheat harvest of the US, which is about 2 billion bushels. (A grain of wheat is about 10 mm^3; a bushel is about 35 liters or 0.035 m^3)

9. How many multiples of 7 are there between 1 and 1000? Can you find the sum of them all?

10. Find the sum 1 + 3 + 5 + · · · + 999.