

MATH 7 HOMEWORK 6: Arithmetic sequences

October 31, 2021

1. Arithmetic sequence (progression)

A sequence of numbers is an arithmetic sequence if the difference between consecutive elements is the same number. This number is called a common difference, d .

For example: 1, 5, 9, 13, 17, ... The difference here is $d = 4$.

Sequence elements (terms) are labeled according to their position in the sequence using a counter n as a subscript. The value of the n -th element in a sequence is labeled as a_n . Then, the first term in the sequence has $n = 1$ and a value of $a_1 = 1$, the second element is $a_2 = 5$, and so on.

We could find any element of a sequence knowing the first element a_1 and the difference d .
For example, what is a_{100} ?

$$\begin{aligned}a_1 &= 1 \\a_2 &= a_1 + d = 1 + 4 = 5 \\a_3 &= a_2 + d = a_1 + 2d = 1 + 2 \times 4 = 9 \\a_4 &= a_3 + d = a_1 + 3d = 1 + 3 \times 4 = 13 \\&\dots \\a_n &= a_1 + (n - 1)d\end{aligned}$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

2. Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Proof:

$$\begin{aligned}a_n &= a_{n-1} + d \\a_n &= a_{n+1} - d\end{aligned}$$

Add the left and the right sides:

$$\begin{aligned}2a_n &= (a_{n-1} + d) + (a_{n+1} - d) \\2a_n &= a_{n-1} + a_{n+1}\end{aligned}$$

Dividing by 2:

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Another property of arithmetic sequences is that we can find the common difference d if we know any 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}$$

3. Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

Proof: we write the sum in 2 ways, in increasing order and in decreasing order:

$$S = a_1 + a_2 + a_3 + \cdots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$$

Adding up left and right sides:

$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

We notice that:

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$$

$$2S = (a_1 + a_n) \times n$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

4. Arithmetic sequences -summary

$$a_n = a_1 + (n - 1)d$$

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

$$d = \frac{a_s - a_t}{s - t}$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$