

MATH 7: HOMEWORK 21
Invariants, and asymptotes
 April 3, 2022

1. Definition for sin and cos of an angle

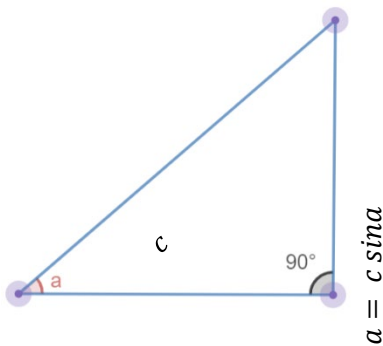
As we discussed, for any angle α , we can find invariants : (sine) $\sin\alpha$ and (cosine) $\cos\alpha$

In general, for a right-angle triangle with hypotenuse not equal to 1, the $\sin\alpha$ and $\cos\alpha$ of the angle are defined as:

$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

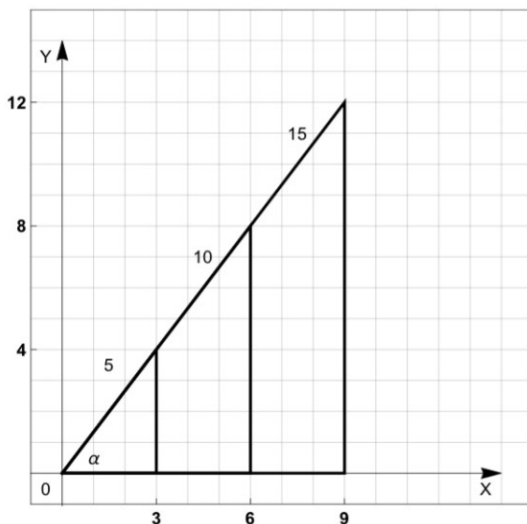
This is because the definitions on \sin and \cos do not really depend on size of the triangle, but only the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle α and hypotenuse c , then the sides will be $c \sin\alpha$ and $c \cos\alpha$:



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{c \sin\alpha}{c}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{c \cos\alpha}{c}$$

Example: Consider the angle α in the following triangles:
 $b = c \cos\alpha$

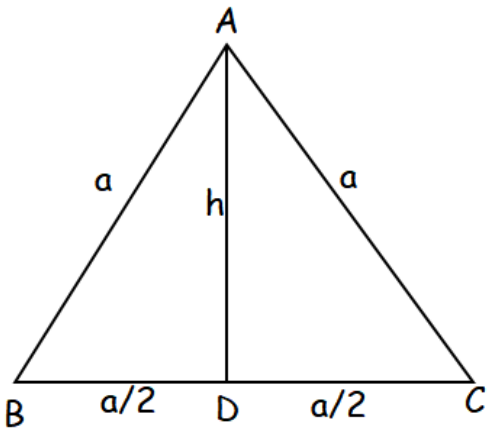


$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

Homework problems

1. As we discussed in class, please find



- $\sin(\angle B)$,
- $\cos(\angle B)$,
- $\sin(\angle BAD)$,
- $\cos(\angle BAD)$

2. Plot these functions, clearly define asymptotes:

a. $y = \frac{1}{x+3} - 3$

b. $y = \frac{1}{3-x} - 3$

c. $y = x - \frac{1}{x}$