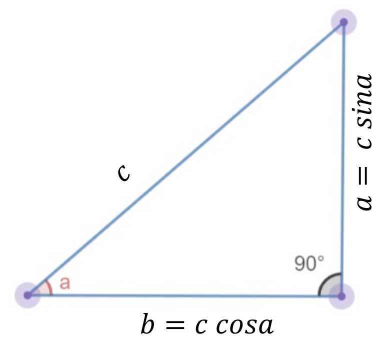
MATH 7: HOMEWORK 24
Trigonometry 3, Radian and trigonometric circle.
May 8, 2022

FIGURE 1.



1. Definition for sin, cos, and tan of an angle

In general, for a right-angle triangle, the ***sin*** α and ***cos*** α of the angle are defined as:

$$\sin(\alpha) = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{a}{c}, \quad \cos(\alpha) = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{b}{c}$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypothenuse}}{\text{adjacent side/hypothenuse}} = \frac{a}{b}$$

2. Radians – new measure of the angle's size

Until now, we have been measuring angles in degrees, which are defined by saying that a full turn corresponds to 360^0. An alternative way to measure angles is by **radians**, which are defined in the following way: given an angle Θ, it's measure in radians is <u>the ratio</u> of the arc (**a**) of circumference with angle Θ by the radius (**r**) of the circumference.

For example, the angle 360^0 corresponds to a full circle. Since the perimeter of a circle is 2 π R, dividing by R gives: $\mathbf{360^0}$ **= 2π rad**. In the same way, half a circle corresponds to an angle of $180^0 = \frac{1/2\times 2\pi\ R}{R} = \pi$ radians. By similar arguments, we can translate all the angles that appeared in our previous table:

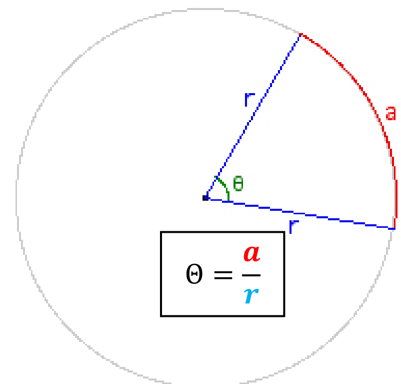


FIGURE 2.

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
		angle in rad	0	π/6	π/4	π/3	π/2
sine	sin(α)	$\frac{\text{opposite side}}{\text{hypothenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	cos(α)	$\frac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	tan(α)	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

3. Trigonometric (unit) circle

A very useful tool in understanding the trigonometric functions is the trigonometric circle (see Figure 3). In order to find the sine and cosine of a positive angle α, we just have to "walk" around the circle a distance α, starting from the point (1; 0) in anticlockwise direction. Then, <u>the coordinates</u> of the point we arrive at are (cos α; sin α). For negative angles α, we define the sine and cosine in the same way, but walking in the clockwise direction.

Note, that you could have angles larger than 360^0. For example, the angle $390^0 = 360^0 + 30^0$ that has the same coordinates on the unit circle as a 30^0 angle (you must circle counterclockwise 1 full revolution and 30 more degrees).

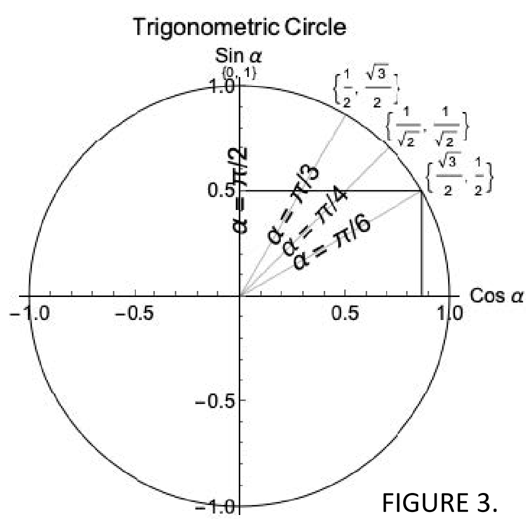


FIGURE 3.

Homework problems

All angles are measured in degrees or radians.

1. Draw a large trigonometric circle. Then, remembering that 2π rad corresponds to a full circle, find the points corresponding to the following angles (write the corresponding letter on the correct point)

a) π
b) $\frac{3\pi}{2}$
c) $\frac{3\pi}{4}$
d) $-\frac{5\pi}{4}$
e) 11π
f) -3π
g) $\frac{25\pi}{3}$
h) $-\frac{19\pi}{6}$

2. Using a calculator, compare the results. Be careful with the settings (DEG or RAD) of your calculator. If the angle is measured in degrees, use DEG. If the angle is measured in radians, use RAD.

a. $\sin 45^0 \quad and \quad \sin\frac{\pi}{4}$
b. $\cos 120^0 \quad and \quad \cos\frac{2\pi}{3}$
c. $\sin 90^0 \quad and \quad \sin\frac{\pi}{2}$
d. $\cos 0^0 \quad and \quad \cos 0$

3. Now, use your trigonometric circle, Figure 3, and the values for sin/cos from table on page 1 to complete this table:

Angle in rad	sine	cosine
π		
3π/2		
3π/4		
-5π/2		
11π		
-3π		
25π/3		
-19π/6		

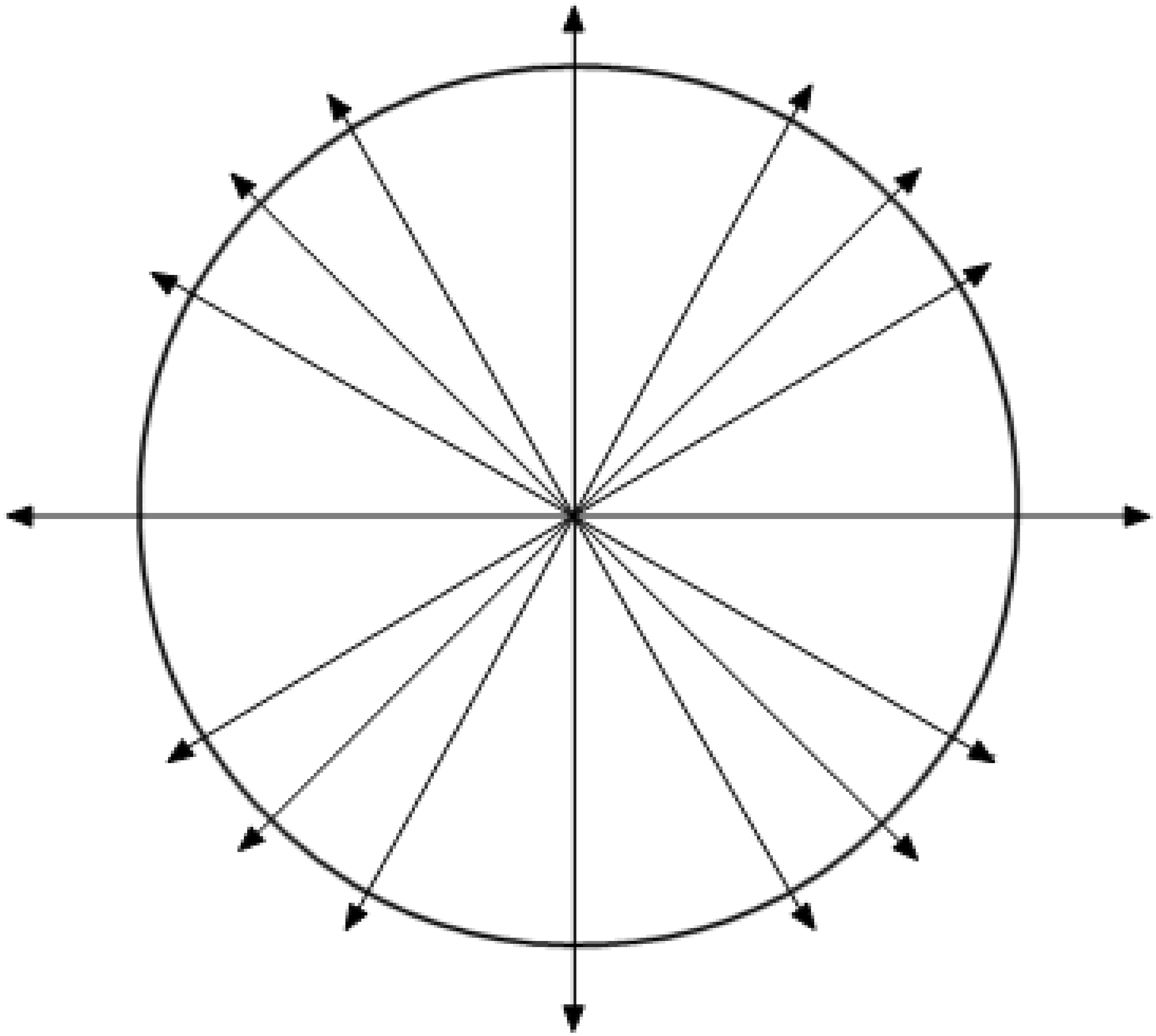
4. (*) Using the trigonometric circle, check if the inequalities are valid for the angles x in the table.

angle $\boldsymbol{x}$	$sin\boldsymbol{x} \geq \frac{\sqrt{3}}{2}$	$\frac{1}{2} < sin\boldsymbol{x} < \frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2} < sin\boldsymbol{x} < \frac{1}{2}$	$sin\boldsymbol{x} \leq -\frac{\sqrt{2}}{2}$
π/7			✓	
2π/7				
-3π/5				
5π/8				
25π/9				

5. Using the trigonometric circle, show that $cos\ x = sin\ (x + \pi/2)$ for any angle x.

6. Find all real numbers x such that $(\sin x)^2 = \frac{3}{4}$ in the interval 0 to 2π. Are there more angles like that outside this interval?

You may find this unit circle helpful for problem 1, or you may draw your own.



This circle may help you relating degrees and radians