Plan Isometries Part 1: Reflection

Definition. A transformation f is an operation which sends every point P of the plane to a new point, f(P).

1. Reflection (symmetry with respect to a line)

Definition. Let *d* be a line in a plane. The reflection with respect to the line d is the plane transformation that associates to any point M, the point M' s.t. *d is the perpendicular bisector of the segment* MM'

The point M' is called the ***image*** de *M or the symmetrical of M with respect to d*

Construction :

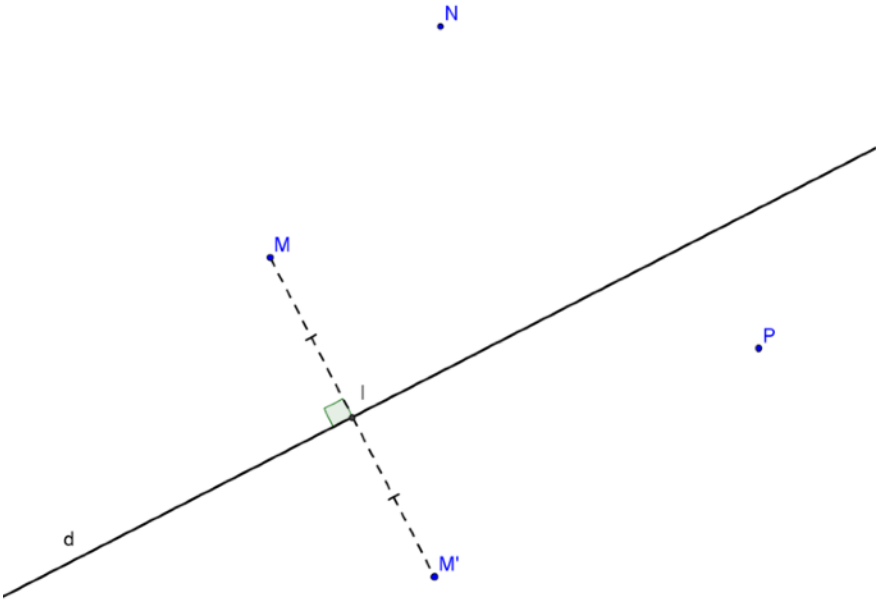


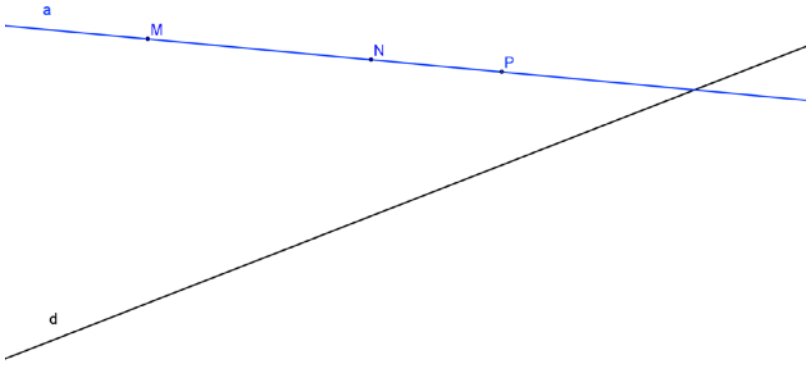
fig. 1

……..

Definition. We say that *M* is ***invariant*** (or ***fixed***) with respect to the transformation *f* of the plane if the image of *M through f is the point itself.*

Properties of a reflection (symmetry with respect to a line):

a) **Conservation of collinearity . The image of a line**



b)

fig. 2

In the figure 2, the points *M*, *N*, and *P* are collinear. Construct the images of *M*, *N* and *P*. What do you notice?

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In the figure 1, which are the reflected images of the points M', N' and P' ?

……..

Observation : The reflection ***with respect to a line*** ***conserves the collinearity of the points.***

Image of a line : What can we say about the intersection of two lines?

Proof (Hint: all the points of a line stay collinear)

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Think of extreme cases (fig 3, 4) : *Construct the image of the line a through a reflection with respect to the line d? What do you observe in each figure?*

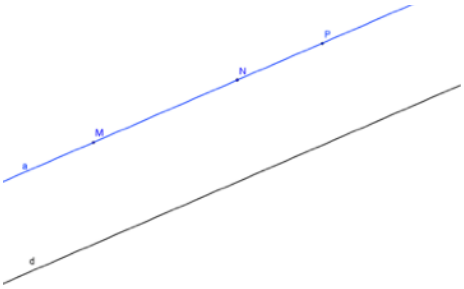
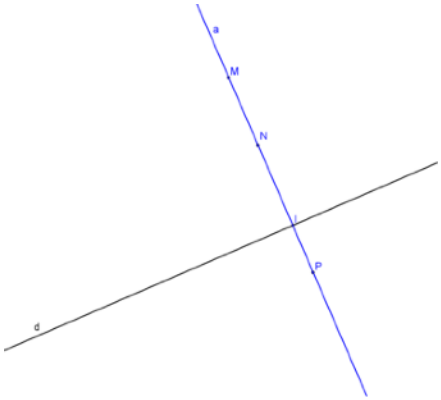


fig. 3

fig. 4



b) Preservation of distances. Image of a segment

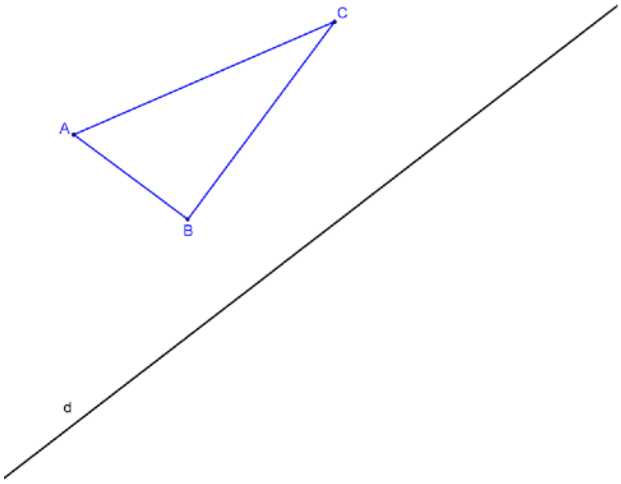


fig. 5

Construct the images of the segments AB , AC and BC through the reflection with respect to the line d.

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What can you say about the length of the segments ?

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Definition. We say that the triangles *ABC* et f(ABC) are s ***isometric*** when the lengths of their sides are equal.

What can you say about the angles of the two triangles?

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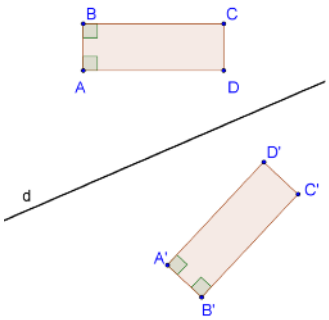
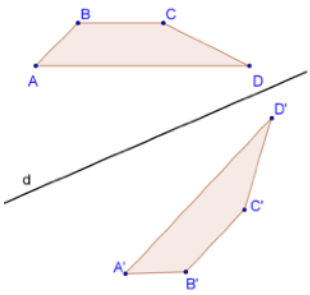
c) Does the reflection with respect to a line preserve angles as well? Can you think of a proof?

Think of extreme cases:

a)The image on a right angle

What properties are preserved when angles are preserved?





d) The orientation is reversed

Intuitively the orientation of a figure is the choice of enumerating its vertices . In the two trapezoids we start in the alphabetical order and we end in the image in the reversed alphabetical order.

An reflection ***does not preserve the orientation of a figure.***

e) Image of a cercle

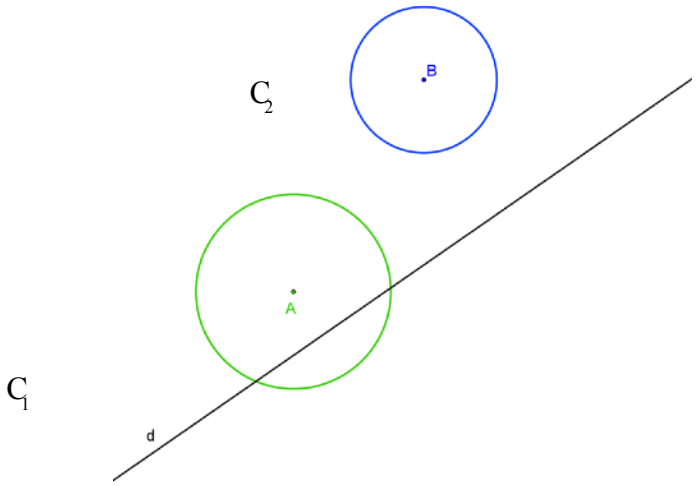


fig. 8

Construct the images of the two circles :

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Can you construct an invariant circle? Where should it be placed?

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……..

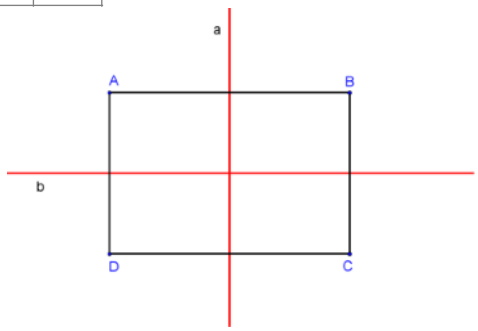
Axis of symmetry

Definition. We say that a line *d* is an ***axis of symmetry*** of a figure F if the figure is ***invariant*** through an reflection with respect to the line *d*.

Examples :

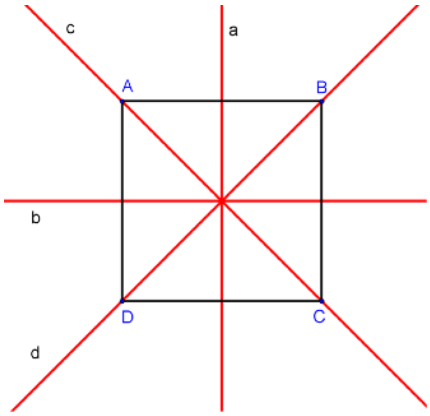
a) A ***rectangle*** has 2 axes de symétrie.

s_a	
A	
B	
C	
D	



s_a	s_b
A	
B	
C	
D	

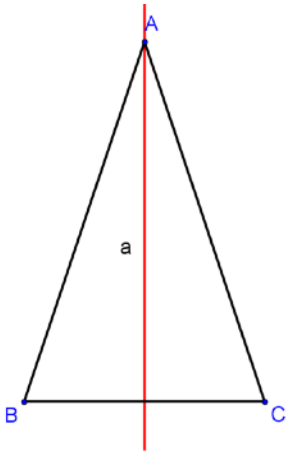
b) A square has 4



s_a	s_c
A	A
B	B
C	C
D	D

s_b	s_d
A	A
B	B
C	C
D	D

c) An ***isosceles triangle*** has one axis of symmetry



S_a	
A	
B	
C	

d) How many axis of symmetry has an ***equilateral triangle ?***
e) How many axis of symmetry has a ***circle*** ?

2. Symmetry with respect to a point

Definition. Let *O* be a point in a plane. The (central) symmetry with respect to the point O is the plane transformation that associates to any point M, the point M' s.t. *O is the middle of the segment* MM'

The point M' is called the ***image*** de *M or the symmetrical of M with respect to O*

Construction :

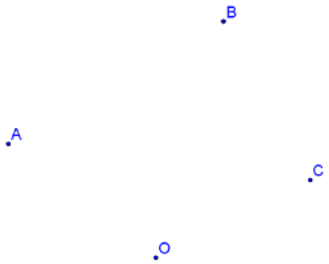


fig. 9

In the figure 9, which are the images of the points A, B, and C ? Which is the image of the point O ?

……..

Observation :*O is the* ***unique invariant point*** of the symmetry with respect to the point O.

In the figure 9, which are the images of the points A', B' and C' ?

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Properties of a symmetry with respect to a point :

a) Conservation of collinearity. Image of a line

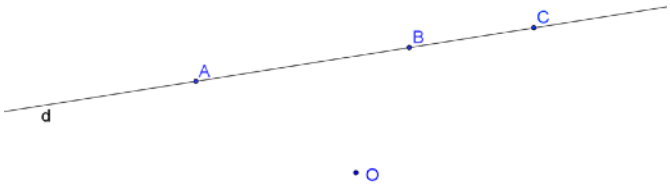

fig. 10

Can you construct a line invariant to the symmetry with respect to a point?

b) Conservation of distances. Image of a segment

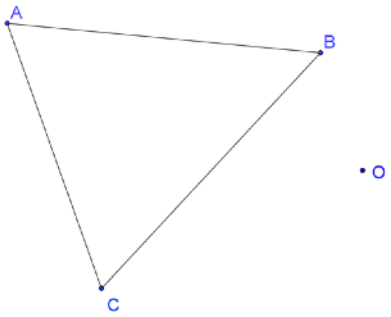

fig. 11

Construct the image of the segments AB, AC, BC

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What can you say about the length of the segments ? What can you say about the angles of the two triangles?

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What geometrical properties are preserved is angles are preserved?

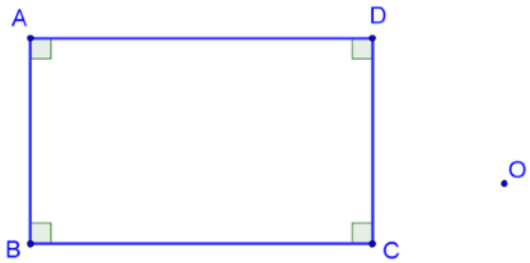


fig. 12

d) Does the symmetry with respect to a point preserve the orientation of a figure?

……..

……..

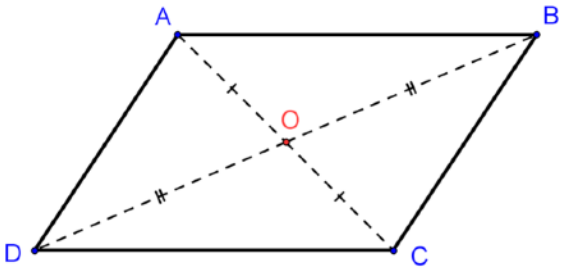
Center of symmetry

Definition. We say that the point P is a center ***of symetry*** of a figure F if the figure is ***invariant*** through a symmetry with respect to the point P.

Examples.

a) A ***parallelogram***

s_O	
A	
B	
C	
D	



b) Does a ***triangle*** have a center of symmetry ? Why or why not?

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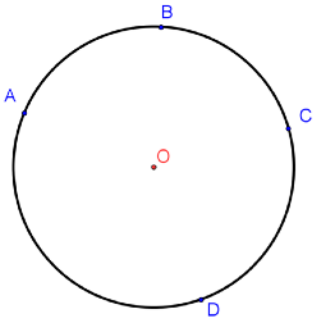
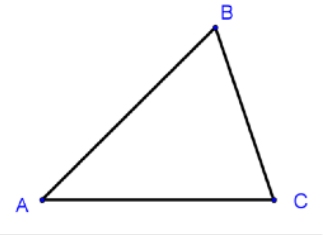
c) Which is the center of symmetry of a ***circle***?





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Homework

1. Let f = Rl be the reflection with respect to the x-axis. For each of the following points P , find the coordinates of the image point f(P): P1 = (1,1) , P2 = (2,3) , P3 = (3,0) , P4 = (5,1)

2. Can you write a general formula: if P = (x, y), then T (P) =??

3. Answer the same questions for reflection with respect to the line x = y.

4. Answer the same questions for reflection with respect to the line x = 1.

5. The image of point $(3,4)$ when reflected in the *y*-axis is

1) $(-3,-4)$
2) $(-3,4)$
3) $(3,-4)$
4) $(4,3)$

6. Point $(-2,3)$ is reflected in the *x*-axis. In which quadrant does its image lie?

1) I
2) II
3) III
4) IV

7. If the point $(2,-5)$ is reflected in the line $y=x$, then the image is

1) $(5,-2)$
2) $(-2,5)$
3) $(-5,2)$
4) $(-5,-2)$

8. The coordinates of point *A* are $(-3a,4b)$. If point A' is the image of point *A* reflected over the line $y=x$, the coordinates of A' are

1) $(4b, -3a)$

2) $(3a, 4b)$

3) $(-3a, -4b)$

4) $(-4b, -3a)$

9. What is the image of point $(-3, -1)$ under a reflection in the origin?

1) $(3, 1)$

2) $(-3, 1)$

3) $(1, 3)$

4) $(-1, -3)$

10. A function, f, is defined by the set $\{(2, 3), (4, 7), (-1, 5)\}$. If f is reflected in the line $y = x$, which point will be in the reflection?

1) $(5, -1)$

2) $(-5, 1)$

3) $(1, -5)$

4) $(-1, 5)$

11. Which transformation of the line $x = 3$ results in an image that is perpendicular to the given line?

12. If $M(-2, 8)$ is reflected in the *y*-axis, what are the coordinates of *M′,* the image of *M*?

13.
Find the image of (1,5) when it is reflected over the line $y = x$.

14. Find the image of $P(2, -5)$ under the transformation $r_{y=x}$.

15. Find the image of $P(4,-2)$ under the transformation $r_{y=x}$.

16. Find the coordinates of the image of point $(5,2)$ after a reflection in the line $y=x$.

17. If point *P* with coordinates (a,b) is reflected in the line $y=x$, what are the coordinates of the image of *P*?

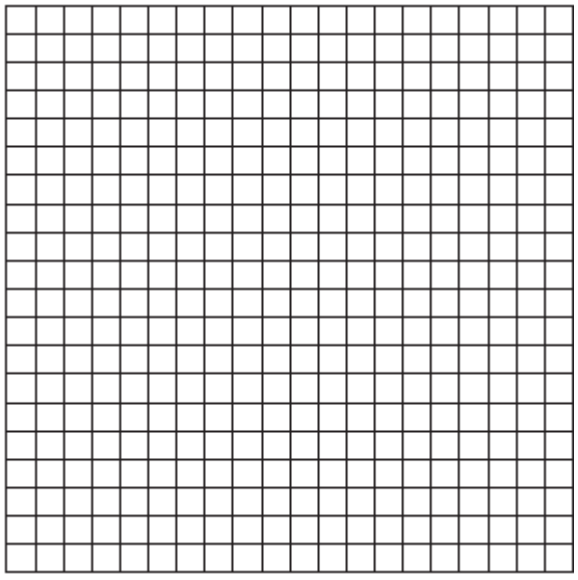
1) $r_{x\text{-axis}}$

2) $r_{y\text{-axis}}$

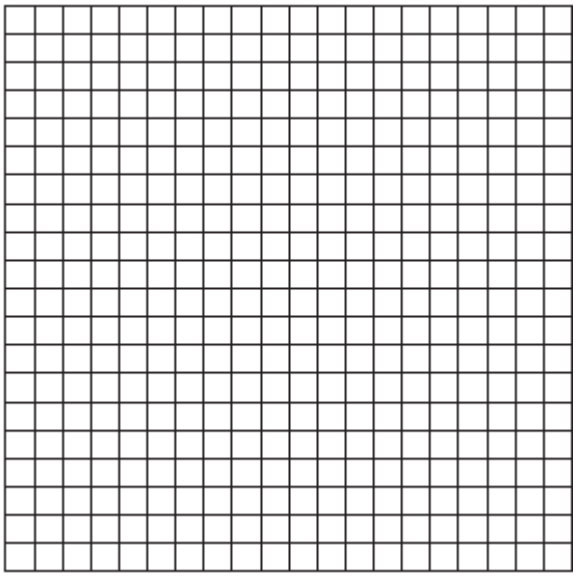
3) $r_{y=x}$

4) $r_{x=1}$

18. Triangle *SUN* has coordinates *S*(0,6), *U*(3,5), and *N*(3,0). On the accompanying grid, draw and label $\triangle SUN$. Then, graph and state the coordinates of $\triangle S'U'N'$, the image of $\triangle SUN$ after a reflection in the *y*-axis.



19. On the accompanying grid, draw and label quadrilateral *ABCD* with points $A(1, 2)$, $B(6, 1)$, $C(7, 6)$, and $D(3, 7)$. On the same set of axes, plot and label quadrilateral $A'B'C'D'$, the reflection of quadrilateral *ABCD* in the *y*-axis. Determine the area, in square units, of quadrilateral $A'B'C'D'$.



20. The coordinates of the endpoints of $\overline{AB}$ are $A(0, 2)$ and $B(4, 6)$. Graph and state the coordinates of A' and B', the images of *A* and *B* after $\overline{AB}$ is reflected in the *x*-axis.