Dilations- Not Isometries

A dilation with center O and a scale factor of k is a transformation that maps every point P in the plane to a point P′ so that the ratio of the distance from the center of dilation to any point on the image compared to the distance from the center of dilation to the corresponding point on the pre-image will result in the scale factor, k.

Definition The dilation of center O and scale factor k, $D_{(O,k)}$ to be the operation that multiplies each coordinate of a point P by a factor k, along the vector $\vec{v} = \overrightarrow{OP} = (x_P - x_O, y_P - y_O)$: a point P is sent to a point P`$=D_{(O,k)}(P) = (x_O, y_O) + (k(x_P - x_O), k(y_P - y_O))$.

Theorem
A dilated line that that does not pass through the center of dilation results in a parallel line.
A dilated line that passes through the center of dilation stays the same.

Why? Dilation constructs similar triangles, leading to parallel lines.

Theorem Triangle Proportionality (Side-Splitter): In a triangle a line intersecting two sides is parallel to the third side iff it divides the first two sides proportionally.

How to determine the center of dilation:

Lines drawn through each point on the pre-image and its corresponding image point will intersect at the center of dilation.

Theorem Dilation is a similarity transformation, i.e. it preserves slopes. As a consequence angle measures, parallelism (things that were parallel are still parallel) and collinearity (points on a line, remain on the line)are preserved.

Sketch of a proof:
Take the simpler case of a dilation with the center in origin. Take two points P,Q. Express the formula of the slope of the line PQ, $m_{PQ} = y_Q - y_P$. Compare it to the the slope of the line P′Q′,
Generalize your proof using Pythagorean theorem, or distance formula for a general translation.

Classwork. Investigate the properties of dilations:

1 Triangle $A'B'C'$ is the image of $\triangle ABC$ after a dilation of 2. Which statement is true?

1) $AB = A'B'$

2) $BC = 2(B'C')$

3) $m\angle B = m\angle B'$

4) $m\angle A = \frac{1}{2}(m\angle A')$

2 If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

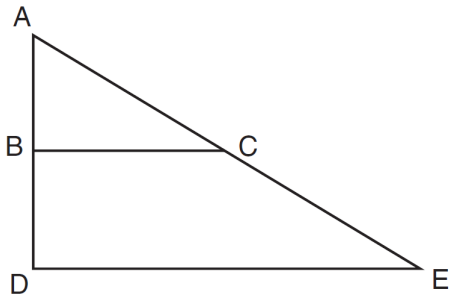
1) $3A'B' = AB$

2) $B'C' = 3BC$

3) $m\angle A' = 3(m\angle A)$

4) $3(m\angle C') = m\angle C$

3 The image of $\triangle ABC$ after a dilation of scale factor *k* centered at point *A* is $\triangle ADE$, as shown in the diagram below.



Which statement is always true?

1) $2AB = AD$

2) $\overline{AD} \perp \overline{DE}$

3) $AC = CE$

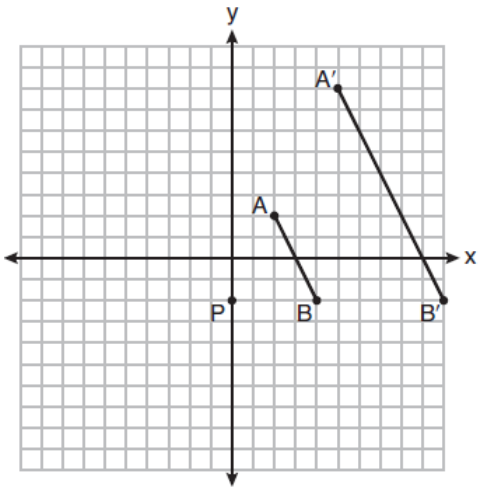
4) $\overline{BC} \parallel \overline{DE}$

4 A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

1) The area of the image is nine times the area of the original triangle.
2) The perimeter of the image is nine times the perimeter of the original triangle.
3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.
4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

Dilations on Cartesian Coordinates

a) On the set of axes below, $\overline{AB}$ is dilated by a scale factor of $\frac{5}{2}$ centered at point *P*.



Which statement is always true?
1) $\overline{PA} \cong \overline{AA'}$ 2) $\overline{AB} \parallel \overline{A'B'}$ 3) $AB = A'B'$ 4) $\frac{5}{2}(A'B') = AB$

b) The line whose equation is $3x - 5y = 4$ is dilated by a scale factor of $\frac{5}{3}$ centered at the origin. Which statement is correct?

1) The image of the line has the same slope as the pre-image but a different *y*-intercept.
2) The image of the line has the same *y*-intercept as the pre-image but a different slope.
3) The image of the line has the same slope and the same *y*-intercept as the pre-image.
4) The image of the line has a different slope and a different *y*-intercept from the pre-image.

c) What is the image of point $A(1, 3)$ after a dilation with the center at the origin and a scale factor of 4?

d) If $P(4, -3)$ is transformed under the dilation D_{-2} with the center at the origin. What is the image of P'?

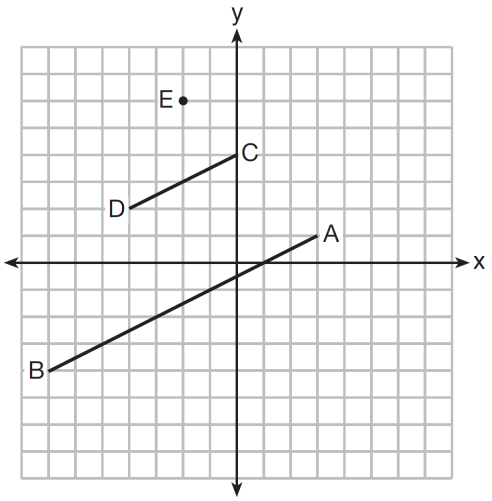
e) If the line represented by $y = -\frac{1}{4}x - 2$ is dilated by a scale factor of 4 centered at the origin, which is the slope and which is the y intercept?

f) A line that passes through the points whose coordinates are $(1, 1)$ and $(5, 7)$ is dilated by a scale factor of 3 and centered at the origin. The image of the line
1) is perpendicular to the original line 2) is parallel to the original line 3) passes through the origin 4) is the original line

g) A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

h) Line segment $A'B'$, whose endpoints are $(4,-2)$ and $(16,14)$, is the image of $\overline{AB}$ after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of $\overline{AB}$?

i) In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor *k* with center *E*.



Which ratio is equal to the scale factor *k* of the dilation?

1) $\frac{EC}{EA}$ 2) $\frac{BA}{EA}$ 3) $\frac{EA}{BA}$ 4) $\frac{EA}{EC}$

j) After a dilation centered at the origin, the image of $\overline{CD}$ is $\overline{C'D'}$. If the coordinates of the endpoints of these segments are $C(6,-4)$, $D(2,-8)$, $C'(9,-6)$, and $D'(3,-12)$. Find the scale factor of the dilation.

Homework:

Dilations with center in the origin of system of coordinates:

S1 If the dilation D_k is an isometry, what must be the value of k?

S2 What are the coordinates of the point $(2,-4)$ under the dilation D_{-2}?

S3 What are the coordinates of point $(-1,4)$ under dilation D_{-2}?

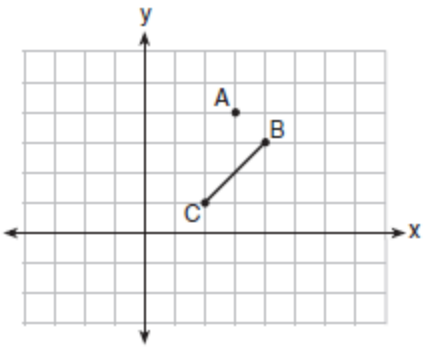
S4 Find the image of $(3,-2)$ under the dilation D_2.

General problems:

H1) After a dilation with center $(0,0)$, the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB = 4.5$ and $D'B' = 18$, find the scale factor of this dilation.

H2) The line represented by $2y = x + 8$ is dilated by a scale factor of k centered at the origin, such that the image of the line has an equation of $y - \frac{1}{2}x = 2$. What is the scale factor?

H3) On the graph below, point $A(3,4)$ and $\overline{BC}$ with coordinates $B(4,3)$ and $C(2,1)$ are graphed.



What are the coordinates of *B'* and *C'* after $\overline{BC}$ undergoes a dilation centered at point *A* with a scale factor of 2?

The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which is the linear equation of its image?

H4)The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which is the linear equation of its image?

H5)The line represented by the equation $4y = 3x + 7$ is transformed by a dilation centered at the origin. Which is the linear equation of its image?

H6)The equation of line *h* is $2x + y = 1$. Line *m* is the image of line *h* after a dilation of scale factor 4 with respect to the origin. What is the equation of the line *m*?

H7) Line *MN* is dilated by a scale factor of 2 centered at the point $(0, 6)$. If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$?

H8) The coordinates of the endpoints of $\overline{AB}$ are $A(2, 3)$ and $B(5, -1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.