

23.1 Homework

You don't have to finish all this completely until next week, but give it a good try and kindly turn it in, showing details of your work. These exercises are just putting together things we have come to know fairly well throughout the year so far, so try exploring them step by step.

1. Consider the line d written as $y = m \cdot x$ for a fixed real value of m and thus any real value x .
 - (a) What is or are the transformation(s) giving the line e written as $y = -m \cdot x$ from line d ?
 - (b) What is the transformation giving the line f written as $y = m \cdot x + b$ with b another fixed non-zero real value, from line d ?
 - (c) What do we obtain if we plot points $P(x, y)$ with coordinates respecting $y = x$ and $x \in [0, 1]$? Also draw it.
 - (d) What do we obtain if we plot points $P(x, y)$ with coordinates respecting either $y = x$ or $y = 0$ or $x = 1$, with $x \in [0, 1]$? Also draw it.
2. Call *the graph of a relationship* what we obtain if we collect all the points $P(x, y)$ respecting some algebraic relationship. Consider the relationship R_C given as $x^2 + y^2 = 1$.
 - (a) What is the geometrical figure of the graph C of R_C ? (If you don't recall at all, try plotting at least eight or ten points, and then try recalling why Pythagoras was useful to convince us what geometric figure C is).
 - (b) What is the relationship (that is, the algebraic expression relating x and y) of the graph C_y , which is a reflection with respect to the Oy axis of the graph C ? Briefly justify.
 - (c) Same question for C_x (Ox axis reflection). Briefly justify.
 - (d) Same question for C_b , the reflection with respect to the line $y = x$. Briefly justify.
 - (e) Which transformation relates C to the graph C_d of the relationship $(x - 1)^2 + y^2 = 1$?
 - (f) What is the graph D of the relationship $(x - 1)^2 + (y - 1)^2 \leq 1$?
 - (g) Which transformation relates D to the graph E of the relationship $(x + 1)^2 + (y - 1)^2 \leq 1$? What points are in the intersection $D \cap E$?
3. Consider a line d given by $y = mx$, a point $P(x_P, 0)$ on the Ox axis and the point $Q(x_Q, y_Q)$ such that Q is the image of P through a reflection with respect to the line $y = mx$. Find the coordinates of Q written in terms of m and x_P . (Hint: recall the geometric way to construct the reflection and write the equations. More hints on the next page).

Detailed steps for exercise 3:

1. Clearly state how we geometrically construct the reflection:
 - (a) we drop the perpendicular h from P on the line d ,
 - (b) we take $\{H\} = h \cap d$ the intersection of d with this perpendicular,
 - (c) we find Q on h such that $QH = PH$.
2. Obtain the first equation by saying that h and d are perpendicular and by writing the slope of h from P and Q .
3. Obtain other necessary equation by noticing that $OP = OQ$ as well (why?) and then writing the squares of their lengths (do not take square roots).
4. Solve the system for x_q and y_q (e.g. by first writing out x_q from the first equation and then substituting its expression in the second one, and clean up).