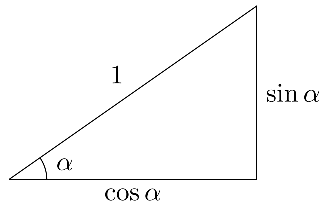
MATH 7

ASSIGNMENT 7: TRIGONOMETRIC RATIOS

DEC 5, 2021

Sine, Cosine and Tangent

For any angle α, we define two numbers, $\sin\alpha$ (sine) and $\cos\alpha$ (cosine) as the lenghts of the legs in the right triangle with hypotenuse 1 and angle α:



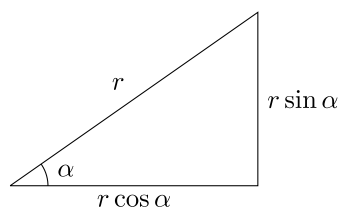
In general, using an arbitrary right-angle triangle with angle α, we can find $\sin\alpha$ and $\cos\alpha$ by the following formulas:

$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$$
$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

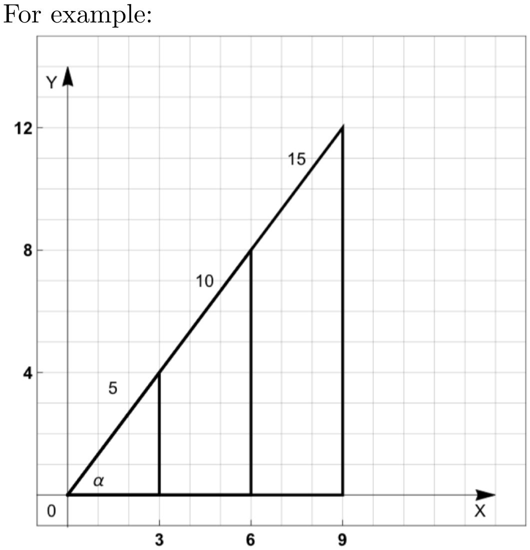
Now we can also define the 3rd trigonometric ratio:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypotenuse}}{\text{adjacent side/hypotenuse}} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Interestingly, the definitions on sin, cos and tan do not really depend on size of the triangle, but only the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle α and hypotenuse r, then the sides will be $r\sin\alpha$ and $r\cos\alpha$:



For example:



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$
$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$
$$\tan\alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{8}$$

There are some special angles for which it is useful to remember the values of sin, cos and tan:

Trigonometric Functions							
Function	Notation	Definition	$0°$	$30°$	$45°$	$60°$	$90°$
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Homework

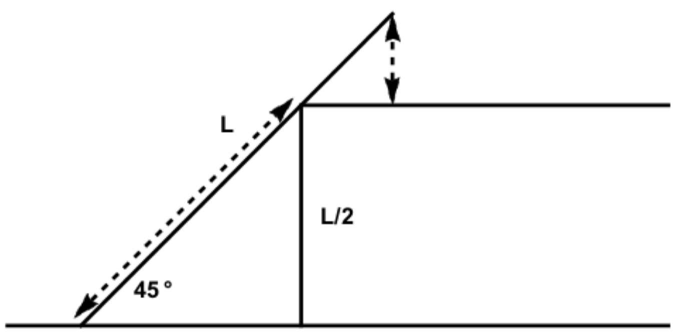
1. Which one is greater?
(a) 0 or $\sin 0°$
(b) 1 or $\sin 30°$
(c) $\sin 45°$ or $\cos 45°$
(d) $\cos 60°$ or $\sin 30°$

2. A tree casts a 60m long shadow when the angle of elevation of the sun is $30°$. How tall is the tree? [Angle of elevation is the angle that line from tip of shadow on ground to top of tree makes with the horizontal.]

3. If a right triangle ΔABC has sides $AB = 3 * \sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, sketch the triangle find all its 3 internal angles.

4. The area of a right triangle is 36 square meters. The legs of the triangle have the ratio of $2 : 9$. Find the hypotenuse of the triangle.

5. A ladder of length L is resting on a ledge whose height is half of the ladder's length. The ladder makes a $45°$ angle with the ground.
(a) How long is the portion of the ladder between the ground and the point of contact of ledge and ladder? [indicated by a long dashed arrow]
(b) At what height is the top of ladder above the ledge? [indicated by short dashed arrow]



6. A cruise ship travels north for 3 miles and then north-west for another 3 miles. How far will it end up from its original position? [North-end is the direction that bisects the angle between north and east.]

*7 Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? [Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.]

*8 Prove that the area of a triangle $\triangle ABC$ can be computed using the formula $A = \frac{1}{2} AB \cdot AC \cdot \sin \angle A$. [Hint: what is the altitude from vertex B?]