MATH 7
ASSIGNMENT 10: PASCAL'S TRIANGLE

JAN 9, 2022

Review of Combinatorics

Today we reviewed some concepts of combinatorics.

- Fundamental Principle of Counting: If the first task can be performed in m ways, and for each of these a second task can be performed in n ways, and for each combination a third task can be performed in k ways, etc. then this entire sequence of tasks can be performed in $m \cdot n \cdot k \cdot ...$ ways.
- Permutations: the choice of k items from a set of n without repetition, and where the order matters. For example, picking first, second, and third place winners from a group of n members, which can be done in $n(n-1)(n-2)$ ways. When $k = n$, there are $n! = n \cdot (n-1) \cdot (n-2) \cdot ... \cdot 2 \cdot 1$ ways. When $k < n$, there are

(1) $$\frac{n!}{(n-k)!} = n \cdot (n-1) \cdot ... \cdot (n-k+1)$$

  ways.
- Combinations: the choice of k things from a set of n things without repetition, and where order does not matter. For example, picking k tv shows from a list of n. There are

(2) $$\frac{n!}{k!(n-k)!}$$

  ways.

Pascal's Triangle

We also discussed the following interesting problem:

How many ways are there to go from the bottom left corner of the chessboard to the upper right, moving always only to the right or up?

This lead us to the following table (we only show part of it):

1	6	21	56	126	252
1	5	15	35	70	126
1	4	10	20	35	56
1	3	6	10	15	21
1	2	3	4	5	6
1	1	1	1	1	1

These numbers are called the *binomial coefficients*. They are usually usually written in a slightly different way:

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ \cdots \end{array}$$

This triangle is called *Pascal's triangle*. Every entry in it is obtained as the sum of two entries above it. The k-th entry in n-th line is denoted by $\binom{n}{k}$, or by ${}_nC_k$. Note that both n and k are counted from 0, not from 1: for example, $\binom{2}{1} = 2$. A very interesting connection between Pascal's triangle and combinatorics is that

(3) $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Homework

1. A dinner in a restaurant consists of 3 courses: appetizer, main course, and dessert. There are 5 possible appetizers, 6 main courses and 3 desserts. How many possible dinners are there?
2. How many ways are there to seat 5 students in a class that has 5 desks? What if there are 10 desks?
3. How many ways are there to select first, second and third prize winner if there are 14 athletes in a competition?
4. A dressmaker has two display windows. The left display is for evening dresses and the one in the right window for regular day dresses. Assuming she can put 10 evening dresses in any order, and separately, 5 regular dresses in any order, how many total possibilities of arranging the two display windows are there?
5. The guidelines at a certain college specify that for the introductory English class, the professor may choose one of 3 specified novels, and choose two from a list of 5 specified plays. Thus, the reading list for this introductory class must have one novel and two plays. How many different reading lists could a professor create within these parameters?
6. How many ways are there to put 8 rooks on a the chessboard so that no one attacks the others?
7. Finish the chessboard problem: how many ways are there to go from lower left corner to upper right corner?
8. Which of the numbers in Pascal triangle are even? Can you guess the pattern, and then carefully explain why it works?
9. What is the sum of all entries in the n-th row of Pascal triangle? Try computing first several answers and then guess the general formula.