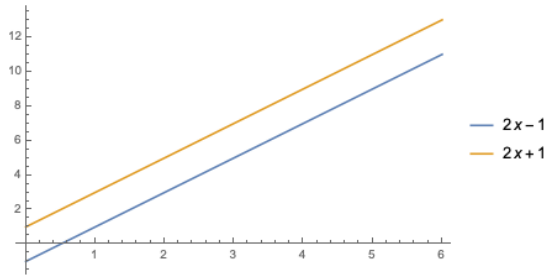
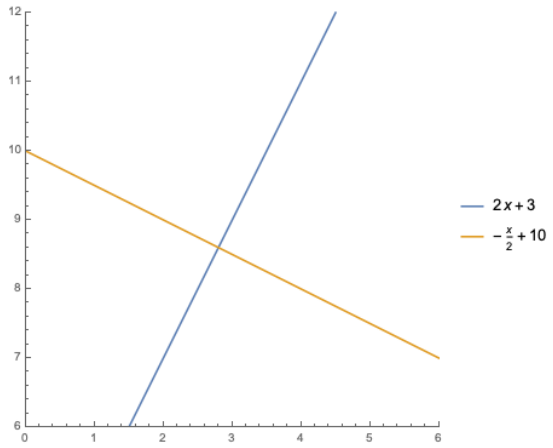
MATH 7
ASSIGNMENT 15: COORDINATE GEOMETRY CONTINUED
FEB 27, 2022

Intersection of two Lines

A general equation of non-vertical line is $y = mx + b$; the number m is called the *slope* of this line. From last week's homework, we learned that, if $m_1 = m_2$, the lines $y = m_1x + b_1$ and $y = m_2x + b_2$ are *parallel*, and as a



consequence they don't intersect. If $m_1 = -1/m_2$, the lines $y = m_1x + b_1$ and $y = m_2x + b_2$ are *orthogonal*, and as a



consequence they intersect at one point, forming a $\pi/2$ rad angle. In the general case where $= 1/m_2 \neq m_1 \neq m_2$, the lines $y = m_1x + b_1$ and $y = m_2x + b_2$ also intersect at one point. Since the intersection point (x, y) belongs to both lines, it should be a simultaneous solution of both equations,

$$\begin{aligned} y &= m_1x + b_1 \\ y &= m_2x + b_2. \end{aligned} \tag{1}$$

Thus finding the point of intersection corresponds to a system of linear equations (1).

Intersection of Line and Circle

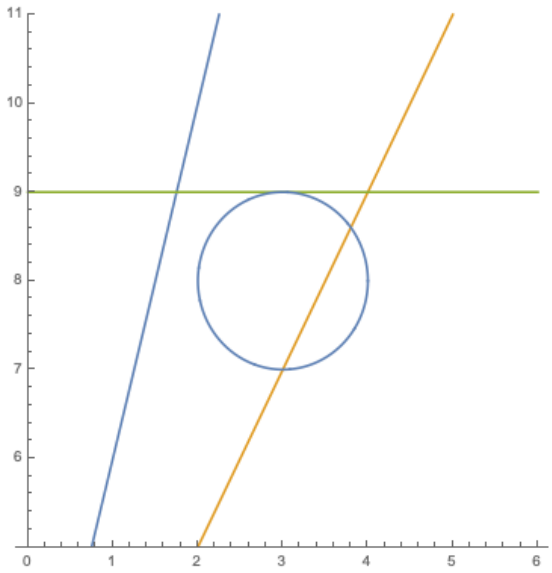
Similarly, the coordinates of a point (x, y) where the line $y = mx + b$ and the circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ should satisfy both equations at once.

$$\begin{aligned} y &= mx + b \\ (x - x_0)^2 + (y - y_0)^2 &= r^2. \end{aligned}$$

Substituting the first equation into the second, we get a quadratic equation in x:

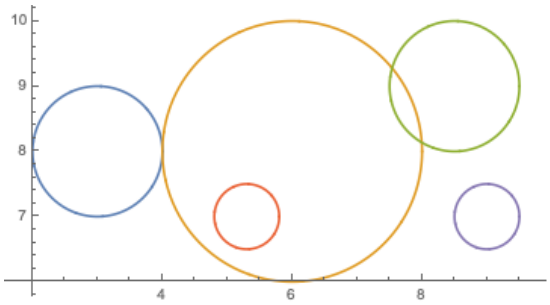
$$(x - x_0)^2 + (mx + b - y_0)^2 = r^2.$$

As we know, this equation might have two different solution, one solution, or no solutions. These cases correspond to a line crossing the circle at two points, a tangent line, and a line not intersecting the circle, respectively.



Intersection of two Circles

In the case of intersection of circles, one can also have that two circles don't intersect, or they intersect in one point (tangent) or in two. In this case, the simplest approach is to find the centers and radii of the circles, and then do a sketch to determine whether there are common points.



Homework

1. Do the lines $y = 2x + 1$ and $y = x + 5$ intersect? Where?
2. What is the equation of the line which intersects $y = 3x + 4$ at the point $(1, 7)$ making 90 degrees?
3. Does the line $y = 2x + 7$ intersect the circle $(x - 2)^2 + (y - 5)^2 = 9$? If yes, where?
4. Consider the line $y = 2x + b$. What are the possible values of b such that the line is:
   (a) Tangent to the circle centered at $(5, 7)$ and of radius 3?
   (b) Intersects the circle centered at $(5, 7)$ and of radius 3 at two points?
   (c) Does not intersect the circle centered at $(5, 7)$ and of radius 3?
5. Find the equations of all the lines passing through $(3, 3)$ which are tangent to the circle of radius 2 centered at $(7, 7)$.
6. Find the centers, radii, and intersection of the circles $x^2-12x+y^2-16y+96 = 0$ and $x^2-16x+y^2-18y+144 = 0$.
7. Consider the line circle $(x - 1)^2 + (y - 2)^2 = 25$ and the circle $(x - 10)^2 + (y - 12)^2 = r^2$. For which values of r does this second circle
   (a) Intersect the first circle at only one point?
   (b) Intersects the first circle at two points?
   (c) Does not intersect the first circle?