MATH 7

ASSIGNMENT 18: MORE PROBLEMS ON GEOMETRY

MAR 27, 2022

Homework

1. Let $A = (1, 0)$, $B = (3, 7)$ and $C = (-1, -3)$. In this problem, you will use two methods to find the point D such that $ABCD$ is a parallelogram.

(a) Sketch these three points in the coordinate plane. Where (roughly) would point D be so that $ABCD$ is a parallelogram?

(b) Find the equation of the line which passes through points A and B

(c) Now find the equation of the line which is parallel to this one (previous item) but which passes through point C. We will call this "line 1".

(d) Now find the equation of the line which passes through points B and C, and then find the equation of the line which is parallel to this one, but passes through point A. We will call this "line 2".

(e) Use the equations of line 1 and line 2 to find their point of intersection.

(f) Sketch points A, B, C, line 1, line 2, and their intersection. Notice that this intersection point is point D. Is it close to your original guess (in part (a))?

(g) In your sketch, show which vectors you can construct from the points A, B and C which, when added, allow you to find point D.

(h) Obtain the components of these vectors (previous item), add them, and find the coordinates of point D. Does your answer agree with the previous method (item (e))?

2. Prove by explicit calculation that $d(R_\phi(x_1, y_1), R_\phi(x_2, y_2)) = d((x_1, y_1), (x_2, y_2))$ for any pair of points (x_1, y_1) and (x_2, y_2) and for any angle ϕ. [Hint: remember that we proved the following trigonometric identity: $(\cos\phi)^2 + (\sin\phi)^2 = 1$ for any angle ϕ.]

Optional

***1.** Consider the circle with center at $C = (x_0, y_0)$ and radius R and the line $y = mx+b$, which is tangent to the circle. Let $A = (x_A, y_A)$ be the point of intersection of the line with the circle. Show that the line passing through C and A is orthogonal to the line $y = mx + b$ defined.