MATH 7
ASSIGNMENT 22: REVIEW TOURNAMENT - TRIGONOMETRY
MAY 1, 2022

Today's battle will review all the main points we covered on dealing with trigonometry.

Homework

1. Consider a right triangle ΔABC, where $B\hat{C}A = 90° = \frac{\pi}{2}$rad is the right angle and we denote the other angles by $\alpha = C\hat{A}B$ and $\beta = A\hat{B}C$.

(a) Express $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\sin\beta$, $\cos\beta$ and $\tan\beta$ in terms of ratios of the lengths for the sides of the triangle, $\bar{AB}$, $\bar{BC}$ and $\bar{CA}$. Doing a drawing might help!

(b) Using the previous expressions, show that the following identities hold:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}, \quad (\sin\alpha)^2 + (\cos\alpha)^2 = 1.$$

(c) From the expressions in A how are the sine, cosine and tangent of β related to the sine, cosine and tangent of α? [One can use these relations to calculate the sine cosine and tangent of 60° from the values at 30°, for example!]

(d) Use all the properties that you learned above to complete the table (for angles between 0 and 90° sine, cosine and tangent have non-negative values):

Trigonometric Functions					
Function	0°	30°	45°	60°	90°
sine	0				
cosine		$\frac{\sqrt{3}}{2}$			
tangent			1		∞

2. George is standing by the river and sees a cluster of bamboo on the other margin. [In this problem you can use the approximations $\sqrt{3} \approx 1.7$ and $\sqrt{2} \approx 1.4$.]

(a) Because he knows the species of bambo, he estimates that the bamboo tree is about 10m high, and the top of the tree makes an angle of about 30° with the ground. Estimate how far away is the bamboo. [Hint: Draw a picture first!]

(b) Then George notices that the line of sight of the trees makes an angle of about 60° with the direction of the river flow. Estimate how wide the river is. [Again, start with a drawing!]

3. (a) Given a triangle ΔABC with sides a, b, and c (where the side a is opposite to the vertex A, etc.), show that the height drawn from vertex A is $h_A = b\sin C$ as in the picture below:

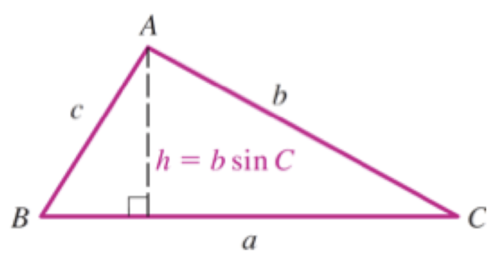


Figure 1. Law of Sines

(b) Likewise show that $h_B = c\sin A$ and $h_C = a\sin B$.

(c) Now use the results above and compute the area of the triangle ΔABC to show the law of sines:

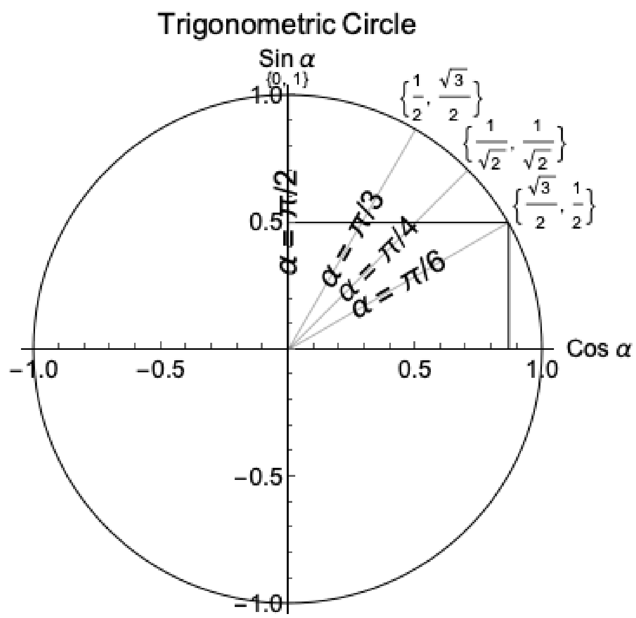
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

(d) Consider an isoceles triangle such that the largest side BC has size 10 and the smallest angle is $A = 120°$. What is the length of the two other sides? [You can use that $\sin 120° = \sin 60°$.]

4. (a) We often consider angles in radians instead of degrees. The are linearly related and $0° \leftrightarrow 0\text{rad}$, $360° \leftrightarrow 2\pi\text{rad}$. Then complete the second row of the table (radians)

Trigonometric Functions												
Degrees	0°	30°	45°	60°	90°	120°	135°	180°	240°	270°	300°	360°
Radians	0											2π
Sine	0											2π
Cosine	0											2π
Tangent	0											2π

(b) Remember that we count angles in the trigonometric circle counterclockwise starting from the positive x axis and use this to complete the next two lines of the table (sine and cosine). You can start by copying down all the information you already have from problem 1(d).



(c) Finally, use the first equation in problem 1(b) to complete the last row (tangent).