MATH 8
HANDOUT 9: CONDITIONALS

Conditional

In addition to all previous logic opeaations, there is one more which we have not yet fully discussed: implication, also known as conditional and denoted by $A \implies B$ (reads A implies B, or "If A, then B"). It is defined by the following truth table:

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Another logic operation is called equivalence and defined as $(A \iff B)$ is true if A, B have the same value (both true or both false).

One can easily see that $(A \iff B)$ is equivalent to $(A \implies B)\,\text{AND}(B \implies A)$.

Also, implication is a logical relationship - it doesn't necessarily mean that A is the reason B is true. For example, you can say "if it is raining, then it is cloudy", written as $(raining) \implies (cloudy)$, and you can take a moment to think about why this makes sense.

Problems

1. Show that $A \implies B$ is not equivalent to $B \implies A$; one of them can be true while the other is false.

2. Prove the contrapositive law: $A \implies B$ is equivalent to $(\neg B) \implies (\neg A)$

3. Show that $(A \implies B)$ is equivalent to $B \vee \neg A$. Can you rewrite $\neg(A \implies B)$ without using implication operation?

4. Consider the following statement (from a parent to his son):
"If you do not clean your room, you can't go to the movies"
Is it the same as:
(a) Clean your room, or you can't go to the movies
(b) You must clean your room to go to the movies
(c) If you clean your room, you can go to the movies

5. English language (and in paricular, mathematical English) has a number of ways to say the same thing. Can you rewrite each of the verbal statements below using basic logic operation (including implications), and variables
A: you get grade A for the class
B: you get score of 90 or above on the final exam
(As you will realize, many of these statements are in fact equivalent)
(a) To get A for the class, it is required that you get 90 or higher on the final exam
(b) To get A for the class, it is sufficient that you get 90 or higher on the final exam
(c) You can't get A for the class unless you got 90 or above on the final exam
(d) To get A for the class, it is necessary and sufficient that you get 90 or higher on the final exam

6. Show that in all situations where A is true and $A \implies B$ is true, B must also be true. [This simple rule has a name: it is called *Modus Ponens*.]

7. Show that if $A \implies B$ is true, and B is false, then A must be false. [This is called *Modus Tollens*.]

8. Use truth tables to show that if $A \implies B$ is true, and $B \implies C$ is true, then $A \implies C$ is also true [This would require a truth table with 8 rows; we will discuss less time-consuming ways of argument next time.]

***9.** (a) Show that $(A \implies B) \implies C$ is not equivalent to $A \implies (B \implies C)$.
(b) Is there any logical relation you could put in place of the star $\star$ in order to make this true?
$((A \implies B) \implies C) = (A \implies (B \star C))$
(c) Is it true that $(A \iff B) \iff C$ is equivalent to $A \iff (B \iff C)$?