MATH 8: HANDOUT 3
COMBINATORICS REVIEW

Please try to do as many of the problems below as you can, and submit your completed solutions through Google Classroom. Some of these problems are similar to those we have discussed in class; some are new. It is OK if you can not solve some problem — but do not give up before making an effort, maybe putting the problem away and coming back to it later — which means you have to start the homework early.

You solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

Classwork

1. A club consisting of 19 people need to choose the chair, the treasurer, and the secretary. In how many ways can they do this?

2. A club consisting of 19 people need to choose one chair and two associates. In how many ways can they do this?

3. In a meeting of 25 people, every one of them shakes hands once with every other. How many handshakes was it altogether?

4. There is a round table seating 10. How many ways there are for 10 people to choose their seats at the table? What if we do not distinguish between two seatings which only differ by rotating the table?

HOMEWORK

Combinatorics:

1. There are four sheep in a pen, each with a distinct color of wool. I want to make a sweater from their wool (I will ask the sheep nicely before shearing them).
   (a) If I want a sweater with two colors of wool, how many possible pairs of colors are there for me to select from?
   (b) If I want a sweater primarily of one color with a trim made of a second color, how many possible ways are there for me to pick these two colors?

2. 12 sentient frogs wish to select 1 leader and then a board of administration. The board is to be comprised of 3 frogs, and the leader may not be on the board. How many possible ways are there to fill the positions? (Assume all the frogs are distinct, with distinct personalities).

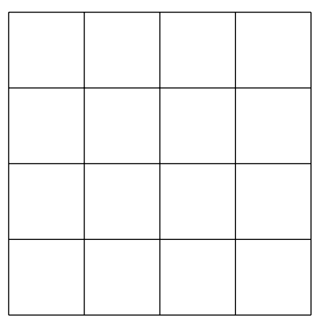
3. How many ways are there to select two black cards and three red cards from a (standard) deck of cards? (In a standard deck there are 52 cards, with exactly half red and the other half black).

4. How many two-digit numbers are there where the first digit is strictly larger than the second digit? Examples: 42, 61, and 10 are such integers, but 18, 44, and 56 are not.
What about three-digit numbers? In this case, count how many there are such that each digit is strictly smaller than the digit to the left.

5. 15 students come to a classroom with 25 seats. How many ways are there of seating these students?

6. How many words one can get by permuting letters of the word "tiger"? of the word "rabbit"? of the word "common"? of the word "Mississippi"?

7. How many different paths are there on 4×4 chessboard connecting the lower left corner with the upper right corner? What about 5×5? The path should always be going to the right or up, never to the left or down.



8. How many "words" of length 5 one can write using only letters U and R, namely 3 Us and 2 Rs? What if you have 5 Us and 3 Rs? [Hint: it is related to the previous problem – each such "word" can describe a path on the chessboard, U for up and R for right. . .]

***9.** You have 10 books which you want to put on 2 bookshelves. How many ways are there to do it (order on each bookshelf matters)?

Miscellaneous Problems:

10. Let a, b, c be distinct positive integers. Is it possible that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$? How about $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$?

11. Let n be a positive integer greater than 10. Is it possible for n to have more than $\frac{n}{2}$ factors?

12. (a) Determine the distance between the two intersection points of the graphs of $y = x^2$ and $y = 2$.
   (b) Determine the distance between the two intersection points of the graphs of $y = x^2$ and $y = x+2$.
   (c) Write the square root of 32 in simplified form as $a\sqrt{b}$ for positive integers a and b (*simplified* means a should be as large as possible).