

Homework for March 6, 2022.

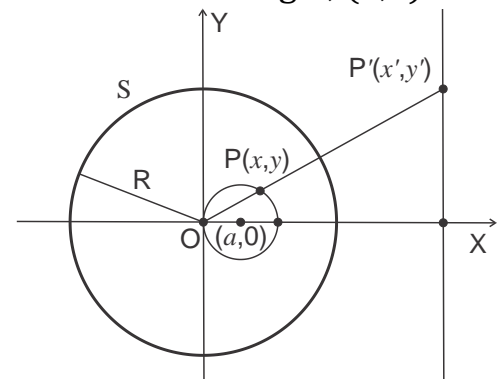
Geometry.

Review the classwork handout on inversion. Solve the unsolved problems from the previous homework. Solve the exercises and the following problems.

Problems.

- Given circle C and its image C' of find the inversion circle, S , which transforms one into another. Consider three cases:
 - circles C and C' are crossing, i.e. have two common points
 - circles C and C' are touching, i.e. have one common point
 - circles C and C' have no common points
- Find the distance between two parallel straight lines that are images of the two circles with the radii r_1 and r_2 , which are tangent at the center O of the inversion circle S with radius R .
- Express the similarity coefficient between circle L and its image L' through radius of the inversion circle R and length of the tangent, $|OT|$. What happens if $|OT| = R$?
- Consider inversion with respect to circle S centered at the origin, $(0,0)$. Image of point $P(x, y)$ is point $P'(x', y')$. Prove that the transformation of coordinates is (see figure),

$$x' = x \frac{R^2}{x^2 + y^2}$$
$$y' = y \frac{R^2}{x^2 + y^2}$$



- What is the image of the line $y = ax + b$?
- Show that in the case $a \neq r$ there exist x_0, y_0, r_0 , such that the image of circle $(x - a)^2 + y^2 = r^2$ is circle $(x' - x_0)^2 + (y' - y_0)^2 = r_0^2$.